Recent Evidence of a Phillips Curve in United States Data: A Re-examination

Ellis W. Tallman
Department of Economics
Oberlin College
10 North Professor Street
Oberlin, Ohio 44074

This Draft: July 2008

Previous drafts circulated under the title "Searching for a Phillips Curve in United States Data." I thank Jim Nason for helpful comments and conversations. Thanks to Thomas King and James Morley for supplying their data and programs as well as to Jaejoon Lee and Charles R. Nelson for making their data and programs available and for their willingness to review the findings. In addition, I thank the participants at the Southern Economics Association meetings November 19-21, 2008 as well as participants in the Atlanta Fed brown bag workshop for suggestions and comments. The views expressed in this paper are those of the authors and not necessarily those of the Federal Reserve Bank of Atlanta, the Federal Reserve Board of Governors, or the Federal Reserve System.

ABSTRACT

This paper re-examines recent empirical work that isolates a Phillips Curve relationship in US macroeconomic data. The empirical methods for each approach are first replicated and then small variations on the estimation are introduced to assess the robustness of the results. The estimations first employ monthly data on unemployment and inflation to highlight the crucial role of estimating the Vector Auto-regression (VAR) model in first differences for the findings and interpretation. Next, the paper employs quarterly data for unemployment, real output, and inflation to generate a model-consistent estimate of the natural rate of unemployment. Using these data as the basis of inquiry, cyclical unemployment estimates (deviations of measured unemployment from the natural rate estimate) display an observable Phillips Curve relationship. These results, like those using monthly data, hinge on whether the data series are stationary in first differences.

Finally, the paper re-examines Lee and Nelson (2007) that uses quarterly data on US unemployment and inflation and that finds evidence supporting a Phillips Curve relationship. I find an alternative set of parameter estimates that generates a higher likelihood than the Lee and Nelson estimates over their 1957Q1 to 2002Q2 data sample, and that provides less conclusive empirical support for a Phillips Curve relationship. Estimates of the model using data up to 2008Q1 generates results with similar properties to this alternative estimate, although a restricted estimate offers support for a Phillips Curve as in Lee and Nelson (2007).

I INTRODUCTION

Macroeconomic research continues to focus on in isolating an observable Phillips Curve relationship in United States data. In various forms, the Phillips Curve illustrates the relationship between inflation and unemployment or between inflation and deviations in the unemployment rate from its natural rate. Recent empirical studies have uncovered an observable short-term, Phillips Curve in both settings.

The Phillips Curve, in its original form as in Phillips (1958), illustrates a negative relationship between the rate of change in nominal wages and the level of the unemployment rate. As it has been often applied in macroeconomics, the Phillips curve was modified slightly to describe a negative relationship between the inflation rate and the level of the unemployment rate. The empirical Phillips Curve as found in US data became a hotly debated issue in the mid- to late 1970s and since because the simple Phillips Curve relationship between inflation and unemployment apparently weakened.[1]

Friedman (1968) and Phelps (1968) offered an interpretation of the Phillips Curve relationship that introduced the concept of the natural rate of unemployment. In their models, the natural rate of unemployment was a structural concept representing the unemployment rate to which the actual unemployment rate would move in the absence of temporary demand disturbances. In this framework, the Phillips Curve relationship reflects a temporary response of real quantities to the demand shocks, and in the long-run, the temporary demand shocks that drive the nominal disturbances are neutral with respect to the real economy.

---

[1] In contrast, Staiger, Stock and Watson (1997) motivate their paper with a graph (their Figure 1, page 35) that illustrates the existence of the negative relationship between inflation and the level of the unemployment rate in the previous year (using annual data 1962-1995).

Recent versions of the Phillips Curve examine movements in inflation (or movements of inflation relative to underlying trend inflation, i.e., cyclical inflation) relative to either output deviations from potential output estimates or to unemployment deviations from a natural rate. Recent empirical studies have uncovered an observable short-term, Phillips Curve by estimating a time-varying natural rate of unemployment and then examining how the deviations of unemployment from the natural rate relate to inflation.

This paper re-estimates the empirical models in King and Watson (1994), in King and Morley (2007), and in Lee and Nelson (2007) all of which find evidence in support of a measurable Phillips Curve. The original results in King and Watson offer support for more than one interpretation of the Phillips Curve, yet their results using reduced form Granger causality tests indicate that lags of unemployment explain subsequent inflation movements, whereas lags of inflation do not explain subsequent movements in unemployment. The discussion of the Phillips Curve correlation in King and Watson (1994) – especially, from which source does the originating shock arise – hinges on the identification assumptions on the contemporaneous influence of unemployment on inflation. In this paper, I highlight that this reduced-form finding depends on the first difference specification and the data sample. Using a larger data sample (adding observations from 1993 to 2007 and from 1948 to 1953), assuming that the inflation series are difference stationary seems debatable.[2] However, under that assumption, the empirical results remain essentially unchanged when the data sample begins at 1954:1 as

[2] Using alternative specifications for the bi-variate VAR, the results can reverse – indicating that lags of inflation explain subsequent unemployment, but not vice versa – or indicate bi-directional Granger causality – that lags of each series can predict subsequent movements of the other.

in King and Watson (1994).[3] Hence, these results are consistent with long-run neutrality of unemployment with respect to inflation-related shocks.

King and Morley (2007) use an alternative approach toward isolating the Phillips Curve in quarterly US data by imposing the restriction of long-run neutrality of inflation-relevant shocks on real quantities like real GDP and unemployment. Their inquiry employs the Vector Auto-regression methodology, and the long-run neutrality restriction is imposed by using identification through long run restrictions. The paper employs data in first differences for the unemployment rate, the log level of real GDP and the log level of the CPI. Hence, the inflation rate is in level form in their specification.

One of the striking findings in King and Morley is their estimate of the natural rate. Previous findings by Staiger, Stock, and Watson (1997) and Salemi (1999) indicate that the natural rate of unemployment in the United States likely moves appreciably over time. King and Morley use a multivariate Beveridge-Nelson decomposition of the same three variables in the VAR specification to estimate a "natural rate of unemployment," defined as the "permanent" component of the unemployment data decomposition. Their results indicate a strong, negative correlation between inflation and deviations of the unemployment rate from their 'natural rate' estimate.

This paper generates the same estimate of the natural rate of unemployment using the empirical methods and data sample period as in King and Morley (2007), and calculates an alternative formulation to estimate the "natural rate of unemployment" using the Vector Auto-regression specification as the basis. The empirical results suggest that the alternative natural rate estimate is largely consistent with the estimate from the

[3] Attempting to isolate the source of the Phillips correlation is deferred to a revision of this paper. Instead, this paper investigates a few empirical results that support the existence (and durability) of the Phillips Curve.

multivariate Beveridge-Nelson decomposition. Adding quarterly observations from 2001Q2 through 2007Q1 (24 observations), re-estimations of King and Morley's model and estimates that use the level of inflation specification generate similar results to the original specification. The multivariate Beveridge-Nelson estimate of the natural rate is not sensitive to whether the specification uses the level of inflation or its first differences. In contrast, the VAR model-based natural rate estimate weakens when the VAR model employs the first difference of inflation.

The King and Morley model is then examined using a monthly data set by adding Industrial Production as a monthly real output measure. The purpose of this exercise is to specify a model with data that are released at a higher frequency so that a real-time measure of cyclical unemployment may be isolated, perhaps for use in monetary policy analysis. The results of the estimation using the monthly data are broadly consistent with the King and Morley results using quarterly data with a few notable exceptions. First, the multivariate Beveridge-Nelson specification is sensitive to whether the model is specified with inflation in levels or first differences in the monthly specification. In addition, the correlation of the VAR based natural rate of unemployment estimate and the multivariate Beveridge-Nelson estimate is lower.

This paper also re-examines the baseline specification in Lee and Nelson (2007) that uses quarterly data on US unemployment and inflation. Their model generates estimates of the natural rate of unemployment and of an underlying trend inflation rate as unobserved components in a state space model and employs a vector auto-regression model to describe the empirical relationship between the associated cyclical movements of inflation and unemployment. Their results indicate strong empirical evidence in support of a Phillips Curve relationship in their baseline model.[4] Using their model and specification, I can replicate the estimation results for their baseline model over their 1957Q1 to 2002Q2 data sample when I use their starting values for the parameter estimates. Starting from a different set of initial conditions, I find an alternative set of parameter estimates with a notably higher likelihood (-88.1 versus -100.22). In the higher likelihood estimate, the relationship between the cyclical time-series components of unemployment and inflation is notably weaker, implying less convincing support for a Phillips Curve in this estimate.

In a further robustness check, I estimate the empirical model in Lee and Nelson with a data sample that extends through 2008Q1. Model estimates using this data set produce results with coefficients and properties that are similar to those found above: namely, the results align with the Lee and Nelson baseline if the estimates start with their initial parameter values, or they align with the alternative parameter estimates if the initial parameter values were those that generated the alternative estimates for original sample. The estimates from the alternative set of initial parameter values have a higher likelihood in both the original and extended samples. The results suggest that the baseline model estimates in Lee and Nelson (2007) were not the global maximum likelihood estimates for that sample, and the estimated properties were also not robust to expanding the data sample to include more recent observations.

---

[4] Lee and Nelson demonstrate that the observation smaller estimates of a Phillips Curve relationship in US data may arise from the use of forward-looking framework as used in a number of New Keynesian Phillips Curve specifications. That is, the one-period ahead forecast horizon for inflation expectations formation will also generate the weakest empirical finding for a Phillips Curve. The dependence of the strength of the Phillips Curve evidence on the forward-looking inflation horizon is the key result in their paper and the results in this paper do not bear on that finding.

In the preliminary new estimates of the Lee and Nelson baseline model, the estimates of the natural rate of unemployment and the trend rate of inflation are notably different from those estimates in the Lee and Nelson (2007). The joint estimates of the model parameters and the long-run trend (and associated cyclical) measures of unemployment and inflation provide a different benchmark for the model implications, especially with respect to the Phillips Curve. Further work on this issue is ongoing.

The empirical results for the King and Watson and the King and Morley models using both monthly and quarterly data suggest that the empirical evidence in support of the Phillips Curve correlation in US data in these papers is robust with respect to the addition of the most recent data to the time series sample. In addition, the findings hold up to whether inflation is specified as stationary in levels or in first differences. The preliminary results for the Lee and Nelson (2007) model seem less sanguine, although the alternative likelihood estimates do not impose standard restrictions like long-run neutrality. Further work will investigate the effects on the estimations of imposing such restrictions.

II MOTIVATION

The Phillips correlation 'resides at business cycle frequencies' (see King and Watson 1994) and that is why King and Watson (1994) as well as others more recently (see Lee and Nelson 2007) isolate the cyclical components of inflation and unemployment to emphasize how the Phillips correlation persists in US data. Like King and Watson, I employ band pass filters to highlight the time-series movements of data series at "business cycle frequencies."[5]

Charts 1 and 2 display six data series each; chart 1 displays data series in the quarterly frequency, and chart 2 displays the monthly data series. For Chart 1, the data series in the left column are: the log level of the unemployment rate, the log level of real GDP, and the log level of the Consumer Price Index. These series in first differences are on the right column, and the data sample runs from 1950Q2 to 2007:Q1. For Chart 2, the left column lists the following monthly data series: the log level of the unemployment rate, the log level of the Industrial Production index, and the log level of the Consumer Price Index. First differences these series are plotted in the right column. The data sample for the monthly series start in 1948M2 to 2007M12.

Chart 3 displays a Phillips Curve correlation and illustrates the time-series properties of cyclical unemployment and cyclical inflation when the monthly data series are filtered using a band pass filter.[6] It is notable that the cyclical inflation rate and the cyclical unemployment rate display substantial increases in the mid to late 1970s and in the early 1980s. These observations and their cyclical patterns are likely crucial for the identification of the Phillips Curve relationships observed in the empirical procedures below.

The contemporaneous correlation between these series is approximately -0.56, and the maximum correlation is -0.65 when unemployment lags inflation by 4 months. These correlation properties are only suggestive because the series have been smoothed

---

5 The appendix in Christiano and Fitzgerald (2003) provides an intuitive motivation and explanation for the use of band pass filters exploiting the frequency domain. The specified filter magnifies the time-series properties within the frequencies between 2 and 8 years.

6 The filter was popularized by King and Watson (1994) and described by Baxter and King (1993).

extensively, but the graph highlights the point emphasizes in King and Watson that the Phillips Curve resides in the business cycle frequency. Chart 4 displays a scatter plot of the cyclical inflation series versus the cyclical unemployment series lagged four months to exploit the maximum negative correlation. In the discussion of King and Morley below, the empirical methods appear to isolate the business cycle fluctuations of the unemployment rate in order to highlight the correlation between these cyclical movements – deviations from a trend or 'natural rate of unemployment' -- and the inflation rate.

III ISOLATING A NATURAL RATE OF UNEMPLOYMENT ESTIMATE

Salemi (1999) introduces a model that incorporates the natural rate of unemployment as an unobserved variable defined within the model's system of equations. The specification of the structural model includes several variables that could explain time variation in a natural rate of unemployment and imposes numerous identifying assumptions about the variables that determine the natural rate of unemployment. Salemi finds ample evidence in support of a time-varying natural rate of unemployment. Plots of the natural rate estimates display notable time-variation in the series and depict several time periods during which the unemployment rate deviated from the natural rate estimate substantially for a number of years. The paper finds that the estimated model contains a Phillips Curve, but the strength of the relationship is perceived as weak.

In contrast to Salemi (1999), King and Morley use alternative time-series techniques to estimate a natural rate measure from a three variable system.[7] King and Morley (2007) estimate the natural rate of unemployment using a multivariate Beveridge-Nelson decomposition. Then, with a set of explanatory variables motivated by economic theory, they then try to explain the time-series movements in the estimated natural rate of unemployment series. The paper tests whether variables that should explain the evolution of a natural unemployment rate series have explanatory power for their natural rate estimates.[8] Movements in the natural rate of unemployment series appear consistent with the predicted influence from the explanatory variables. Also, they find that measures of cyclical unemployment (defined as deviations from the natural rate of unemployment) are notably negatively correlated with inflation. The central findings in the paper highlight that there is a significant, short-term Phillip Curve relationship in US data over the period 1948 to 2001, when using the cyclical unemployment measure.

Lee and Nelson (2007) estimate both expected inflation and the natural rate of unemployment within an unobserved components model, allowing for independent stochastic trends and related cycles. The paper emphasizes that the unobserved components method provides a flexible empirical strategy to isolate the Phillips Curve in US data. Their results support the existence of the Phillips Curve, and their estimates highlight how the expected inflation measure tracks closely actual measured inflation as

---

7 King and Morley (2007) interpret the natural rate of unemployment as the steady state unemployment rate consistent with the long run level of aggregate production. They separately regress their natural rate estimate on the key factors that determine labor market equilibrium, like population growth, demographic characteristics of the labor force, labor force participation, education level of the population, etc. They use this regression to suggest that fundamentals underlie the movements in their natural rate estimate.

8 This characteristic distinguishes the methodology in King and Morley (2007) from Salemi (1999).

an outcome of the procedure.[9] The contrast is that the natural rate estimate seems relatively flat. The implied cyclical unemployment measure – deviations of unemployment from the natural rate -- then is much more persistent than other estimates, especially in comparison to the measure of cyclical unemployment in Chart 3. This idea is explored further below.

The unobserved components approach to empirical work on the Phillips Curve can be exploited in real time. Output gaps and cyclical unemployment measures are used in practice for monetary policy analysis; any model that can be updated and used in real-time to analyze the state of the real-sector gaps is desirable. Further work on these models may be promising.

IV REPLICATING REDUCED FORM RESULTS IN KING AND WATSON

King and Watson (1994) generate reduced-form results for the relationship between unemployment and inflation using a bi-variate vector auto-regression. The model is specified in first differences, includes 12 months of lags, and is estimated over the sample period 1954:1 to 1992:4. Over this sample period, both unemployment and inflation are stationary in first-differences. Table 2 displays the results for augmented Dickey Fuller tests for unit roots in data series over the King and Watson sample period as well as expanded samples.[10]

---

[9] Harvey (2007) adds an unobserved random walk component to a single equation Phillips Curve model to account for the "underlying level of inflation" in the model. The paper suggests that to identify a Phillips Curve, it seems necessary to "detrend" inflation rather than to first difference the series. The research expands on work in Kuttner (1994), which uses inflation to refine the estimation of potential output. Harvey estimates a Phillips Curve model using an output gap and trend inflation measures that are identified in the unobserved components model.

[10] It is notable that the classical unit root tests suggest that inflation may not be non-stationary over the sample period 1950:2 to 2007:12, suggesting that expanding the sample time-series to include

Table 3 displays the Granger Causality tests for the bi-variate VAR. The results for the King and Watson specification over the extended sample 1954:1 to 2007:12 indicate that the original inferences from the sample 1954:1 to 1992:12 remain essentially unchanged. Although the classical augmented Dickey-Fuller tests indicate that inflation may be stationary over the full-sample, it is notable that the introduction of the Korean War data observations may be responsible for the findings. Without more intensive investigation, the results suggest that the King and Watson results remain intact.

V ESTIMATING THE STRUCTURAL VAR MODEL

King and Morley (2007) estimate a structural VAR model with three macroeconomic time-series of US data: the unemployment rate, the natural log of real GDP and the natural log of the Consumer Price Index. As mentioned earlier, Chart 1 displays the data series under study in levels and in differences from 1950:2 to 2007:1. Notice that the log level of the price level is a variable in this model, not the inflation rate. Hence, King and Morley assume that inflation is stationary in levels.[11]

The VAR model employs the time-series in first-difference form to yield the following stationary model:

$$\Delta x_t = c + \sum_{k=1}^{K} F_k \Delta x_{t-k} + e_t, \; t = 1, \cdots, T \tag{1}$$

---

observations from the Korean War along with the recent "Great Moderation" since the mid-1980s altered the inference about the time series properties of inflation. In contrast, the results for the unemployment rate suggest that it remains a difference stationary series over the expanded sample. The results of the unit root tests suggest that, at least for the full sample period, the reduced form regressions should be estimated using the difference of unemployment and the levels of inflation.

[11] King and Morley (2007) do not test explicitly whether their data series are stationary, although from the results from replicating King and Watson, it is likely that including the Korean War data suggests that inflation is stationary.

where Δx_t denotes a 3×1 vector of current dated observations for period *t* on the three variables in the VAR (the change in the unemployment rate, the quarterly rate of growth in real GDP, and the CPI measured quarterly inflation rate); the F_k are 3×3 coefficient matrices; and c is a 3×1 vector of constant terms. The error term is assumed to be a Normal and independently distributed 3×1 vector such that $E[\varepsilon_t \mid \Delta x_{t-s}, s > 0] = 0$, and $E[\varepsilon_t \varepsilon_t' \mid \Delta x_{t-s}, s > 0] = \Sigma > 0$ for all *t*.

To identify the VAR, King and Morley employ long-run restrictions on the relationship between the observed data and the identified shocks.[12] The long-run restrictions impose the long-run neutrality of shocks related to the inflation rate – nominal shocks, perhaps related to monetary policy – on the real quantities of real output and unemployment. That restriction introduces two zero restrictions and provides two of the three restrictions required to just-identify the VAR model. The additional restriction imposes a long-run neutrality of shocks associated with real output on the unemployment rate. The motivation for this long-run neutrality assumption is less obvious, however, it can be motivated by suggesting that shocks to real output (supply shocks) have only a temporary effect on unemployment separate from those shocks that affect the natural rate of unemployment directly.

The structural model in an infinite-order moving average representation is:

$$\Delta x_t = m + \sum_{k=1}^{\infty} A_k v_{t-k} \qquad (2)$$

12 See Blanchard and Quah (1989).

where m is a vector of deterministic drifts for the level variables in x_t, A_k is a matrix of shock coefficients and v_t is a vector of three structural shocks. Standard assumptions of zero mean, unit variances and zero cross-correlations apply to the shocks. In addition, the shock series are assumed to be stationary.

For identification, the following assumptions are imposed:

$$\sum_{k=0}^{\infty} a_{31,k} = 0, \qquad \sum_{k=0}^{\infty} a_{32,k} = 0, \quad \text{and} \quad \sum_{k=0}^{\infty} a_{12,k} = 0$$

where $a_{ij,k}$ is the i,jth element of A_k. Restricting the sum of the coefficients to zero implies that shocks associated with the log of real GDP (variable 1) have no long run effect on the unemployment rate (variable 3), that shocks associated with log CPI (variable 2) have no long run effect on the unemployment rate or on the log of real GDP. The shock associated with the unemployment rate (variable 3) is unrestricted, and hence may have a permanent effect on the unemployment rate as well as on other series.

King and Morley offer a standard interpretation of the identified shocks as aggregate supply (shock associated with log real GDP), aggregate demand (shock associated with log CPI) and natural rate shock (shock associated with unemployment rate). The estimated structural VAR generates variance decompositions and impulse responses that are consistent with these interpretations of the shocks.[13]

Chart 5 displays impulse response functions for the unemployment rate, the log level of real GDP and the inflation rate, when the model employs the inflation rate in levels. Note that the shock related to the unemployment rate has a permanent effect by raising the level of the unemployment rate, a permanent negative effect on the log level

[13] King and Morley's VAR results for the sample ending in 2001Q1 have been duplicated, but they are not included because those results are not the focus of the paper.

of real GDP, and a temporary, positive effect on the inflation rate. This temporary effect of the unemployment-related shock on the inflation rate seems to last only two quarters. The last effect is small, but it suggests an effect contrary to the Phillips Curve intuition. The shock associated with the inflation rate, though, displays characteristic correlations often associated with the Phillips Curve. As noted before, the VAR specified with inflation in levels suggests that the shock associated with inflation has only a temporary effect on the inflation rate, although the temporary effect persists nearly 10 quarters. In addition, a positive shock associated with inflation generates a notable negative response in the unemployment rate that seems to persist for about four quarters.

One interpretation of these results is that the long-run neutrality of inflation shocks on real quantities emphasizes the short-run influence that inflation-related shocks may have on unemployment and real output. The model results indicate responses consistent with an operating Phillips Curve without displaying a Phillips Curve correlation explicitly. Chart 6 shows the impulse responses from a VAR specified in first differences of inflation; the first differenced inflation reduces the explanatory power of the inflation-related shock on unemployment and real output. Hence, the assumption that inflation is stationary in levels leads to evidence consistent with a Phillips Curve, and a model with inflation in first differences would be less favorable.

VI ESTIMATING A NATURAL RATE OF UNEMPLOYMENT

King and Morley estimate a natural rate of unemployment as the time-varying permanent component of a multivariate Beveridge-Nelson decomposition that uses the same three time-series as in the VAR. If the natural rate is viewed as the time-varying steady state of unemployment, King and Morley (page 552) suggest that the estimate of the natural rate of unemployment is independent of the structural model underlying the VAR as long as the reduced-form model is correct. In contrast, the structural model underlying the VAR can be used to isolate an alternative estimate of the natural rate of unemployment. One intuitively appealing estimate is to accumulate the permanent shocks associated with the natural rate of unemployment. By construction, these shocks determine the underlying trend movements in the unemployment rate, and can be considered permanent movements in the unemployment rate.

Chart 7 plots the natural rate of unemployment time-series estimated using the multivariate Beveridge-Nelson (MVBN) decomposition against the alternative natural rate generated from the accumulation of the permanent shocks associated with the unemployment rate in the structural VAR. The two natural rates of unemployment measures display essentially the same time-series patterns. Among the small differences, the MVBN natural rate moves sharply at short intervals, whereas the SVAR appears to be a slightly smoother time-series. The MVBN natural rate is the permanent component of the decomposition, which is by assumption a random walk process. The SVAR natural rate estimate need not display that characteristic, but in this application, the natural rate estimate appears to have one.

Chart 8 displays the time series of the deviations of the unemployment rate from the natural rate estimates along with the inflation rate. The negative contemporaneous correlation between these two cyclical unemployment series and inflation is obvious, and this data pattern is observable in the Phillips Curve scatter plots in Chart 9. The scatter plots show the negative correlation between inflation and the two alternative cyclical unemployment measures (as deviations from the estimated natural rate). It is notable that estimates of the natural rate from either model (VAR or MVBN) can produce a similar, negative correlation with inflation when specified with inflation in differences.

VII A SHORT-RUN PHILLIPS CURVE USING MONTHLY DATA

The results from the King and Morley exercise to generate a natural rate of unemployment using a three variable VAR model provide motivation to apply that model to monthly data. Chart 10 displays the impulse response functions from the estimated VAR model using the first differences of the unemployment rate, the log of industrial production, and the log of the Consumer Price Index. The inferences from the quarterly model appear to hold in the monthly model; the shocks associated with the unemployment rate have a negative and permanent effect on the log of industrial production and a positive and short term effect on the inflation rate. The shock associated with inflation has a temporary (nearly twenty four months) negative effect on the unemployment rate and a positive and temporary (of comparable duration) effect on the log of IP.

As with the quarterly VAR, the data series can be used to generate estimates of the natural rate of unemployment; I produce two estimates, one using the multivariate Beveridge-Nelson decomposition and one that accumulates the shocks associated with the unemployment rate. The series produce charts that closely mimic the plots in Charts 7, 8, and 9. Chart 11 displays the two cyclical unemployment measures versus the monthly inflation rate, and suggests that the same dynamics that operated on a quarterly basis are also observable in monthly data. Chart 12 displays scatter plots of the respective cyclical unemployment measures with the inflation rate, again consistent with the findings in quarterly data.[14]

Discussion

The specification of the VAR and the characteristics of the multi-variate Beveridge-Nelson decomposition assume the data series are I(1) in levels. The differenced data, especially the unemployment and inflation in Chart 1, display distinct periods in which the volatility of both series increase substantially. These high volatility periods appear in the early 1950s and the late 1960s through the mid-1980s. In addition, there are periods of relative increase in volatility at business cycle peaks and downturns.

The short-term nature of the Phillips Curve relationship in this setting is also worth investigating. Salemi (1999) notes that the underlying forces that determine the natural rate of unemployment are likely to be slow-moving and long-lived, like demographics. The motivating chart in Staiger, Stock and Watson (1997) displays a Phillips Curve between the inflation rate over a year and the unemployment rate in the previous year. It is worth reconsidering the time-series characteristics that we expect in a natural rate of unemployment estimate, and whether these different methods deliver estimates with those characteristics.

The construction of the cyclical unemployment rate appears to magnify the correlations between that measure and the inflation rate. In the SVAR case, the construction of the natural rate of unemployment accumulates the permanent shocks associated with the unemployment rate (interpreted as the natural rate shock).

---

[14] Note that the model specifications with monthly data that used inflation in first differences failed to generate the strong, negative correlations between the estimated cyclical unemployment rate and the inflation rate as was observed in the quarterly data.

Differences between this measure and the unemployment rate are then by construction some combination of the transitory shocks associated with inflation (interpreted as the aggregate demand shock) and of the permanent shocks associated with real output (interpreted as the aggregate supply shock). Table 4 lists the variance decomposition results for the structural VAR. For the full-sample, the evidence indicates that the unemployment rate variance is mainly explained by the permanent natural rate shocks (64 percent at the 4 quarter horizon) and by the aggregate demand shock (30 percent at the 4 quarter horizon). Also, the aggregate demand shock explains nearly 70 percent of the variation in the inflation rate by the fourth quarter horizon. Future work will try to assess the sensitivity of the results to first differences in the inflation rate.[15]

VIII REPLICATING LEE AND NELSON (2007)

Using the same empirical model estimated in Lee and Nelson (2007), this section generates estimates of three concepts: 1) the "natural rate" of unemployment, or the "non-accelerating inflation rate of unemployment" (NAIRU), 2) the trend rate of inflation, and 3) the estimated Phillips Curve correlation between cyclical (or transitory) deviations from these concepts. As noted above, the estimation of the "natural rate" of unemployment involves the isolation of an unmeasured quantity, defined by its relationship with the observed unemployment rate and the observed relationship between cyclical deviations of unemployment and inflation from the long-run concepts. The

[15] The monthly application of the VAR model is motivated by the goal of employing the output from the model as an input to monetary policy analysis. It would be helpful to uncover whether the monthly model and the associated empirical results can aid in analyzing the real-time policy problem. Further work will focus on designing a quasi-policy setting to evaluate the model in such applications.

concentration on the correlation between the two cyclical entities of these measures differentiates the Lee and Nelson approach from King and Morley examined above.

Results concerning the Phillips Curve correlation are of interest for policy making as well as for verifying the empirical reliability of the proposed relationship. However, the resulting estimates of the underlying series of interest – the natural rate of unemployment and the underlying rate of inflation – may vary notably across the specifications. It is important to examine the time-series properties of these estimated series, discuss whether the observed properties are reasonable given what the time-series is meant to measure, and how the joint estimation of these series and the results on the Phillips Curve accord with the predictions of the standard expectations-based Phillips Curve. The model description is drawn entirely from Lee and Nelson (2007). First, it is required (and then verified in both series for both samples) that each series – quarterly CPI inflation rate and the quarterly unemployment rate – is stationary in first differences. Second, it is assumed that each series can be modeled as unobserved components – the sum of a long-term trend that is a random walk and a stationary transitory component. Finally, the time-series relationship between the two cyclical series (i.e., the transitory deviation from the long-term trends) can be modeled as a vector auto-regression. The specification is outlined below:

Let π_t represent the measured inflation rate
π_t^* represent the trend inflation rate
π_t^c represent the cyclical inflation rate
Let u_t represent the measured unemployment rate
u_t^n represent the natural rate of unemployment
u_t^c represent the cyclical unemployment rate

$$u_t = u_t^n + u_t^c$$
$$u_t^n = u_{t-1}^n + v_{u,t} \ \ where \ \ v_{u,t} \sim i.i.d.N(0,\sigma_{v_u}^2)$$
$$u_t^c = \beta_0 \pi_t^c + \sum_{i=1}^{p_3} \rho_i u_{t-i}^c + \sum_{i=1}^{p_4} \beta_i \pi_{t-i}^c + \varepsilon_{u,t}$$
$$where \ \varepsilon_{u,t} \sim i.i.d.N(0,\sigma_{\varepsilon_u}^2)$$

The p_j, j=1,2,3, and 4 denote the number of lags of unemployment and inflation in the equations. It is also assumed that all disturbances, $v_{u,t}, \varepsilon_{u,t}, v_{\pi,t}, \varepsilon_{\pi,t}$, are serially uncorrelated. The innovations from the structural VAR model, $\varepsilon_{u,t}$ *and* $\varepsilon_{\pi,t}$, are uncorrelated by assumption, so that any correlation between u_t^c *and* π_t^c comes from the contributions picked up in the non-zero values of the parameters, α_0 *or* β_0.

Following Lee and Nelson, I assume that the lag length is 2 and the lagged values of u_t^c *and* π_t^c are assumed to pick up any dynamic relationship between these series. Lee and Nelson (page 166) show that under a certain identifying assumption, the structural VAR can be reduced to a standard VAR. From a state space modeling perspective, the transition equation becomes estimable in an unobserved components model setting.

Let $Y_t = (u_t, \pi_t)', \mu_t = (u_t^t, \pi_t^*)', \psi_t = (u_t^c, \pi_t^c)', V_t = (v_{u,t}, v_{\pi,t})', and\ \zeta_t = (\varepsilon_{u,t}, \varepsilon_{\pi,t})'$.

Rewriting the system in the matrix form:

The unobserved components part is:

$$Y_t = \mu_t + \psi_t$$
$$\mu_t = \mu_{t-1} + V_t \ , \ \mathrm{cov}(V_t) = \begin{pmatrix} \sigma_{v_u}^2 & \sigma_{v_u, v_\pi} \\ \sigma_{v_\pi, v_u} & \sigma_{v_\pi}^2 \end{pmatrix}$$

The structural VAR is then:

$$\Phi_0\psi_t = \Phi_1\psi_{t-1} + \Phi_2\psi_{t-2} + \zeta_t$$

$$where\ \Phi_0 = \begin{bmatrix} 1 & -\beta_0 \\ -\alpha_0 & 1 \end{bmatrix} \quad \Phi_k = \begin{bmatrix} \rho_k & \beta_k \\ \alpha_k & \phi_k \end{bmatrix} \ k = 1,2$$

We need an identifying assumption on the contemporaneous correlation between u_t^c *and* π_t^c to identify the model econometrically. Practically speaking, that boils down to assuming that either α_0 *or* β_0 equals zero. Lee and Nelson adopt a Choleski ordering in which unemployment affects inflation, but not vice versa, so that $\beta_0 = 0$.

The reduced form model is:

$$\psi_t = \sum_{i=1}^{2} \Phi_i^* \psi_{t-i} + e_t$$

$$where\ \Phi_1^* = \Phi_0^{-1}\Phi_1 \ and\ \Phi_2^* = \Phi_0^{-1}\Phi_2 \ for\ i = 1,2\ and$$

$$\text{cov}(e_t) = \Phi_0^{-1}\text{cov}(\zeta_t)\Phi_0^{-1'}$$

To estimate the reduced form of the model, the assumption that $\beta_0 = 0$ makes the matrix (the inverse): $\Phi_0^{-1} = \begin{pmatrix} 1 & \alpha_0 \\ 0 & 1 \end{pmatrix}$

Lee and Nelson present the unobserved components model and the standard form of the VAR equations in a state-space form and estimate the parameters of the system with maximum likelihood methods using the Kalman filter.

Measurement equation: $Y_t = \Gamma B_t$

$$\begin{pmatrix} u_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 1\ 1\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 1\ 1\ 0 \end{pmatrix} \begin{pmatrix} u_t^n \\ u_t^c \\ u_{t-1}^c \\ \pi_t^* \\ \pi_t^c \\ \pi_{t-1}^c \end{pmatrix}$$

Transition equation: $B_t = F\,B_{t-1} + \xi_t$

$$\begin{pmatrix} u_t^n \\ u_t^c \\ u_{t-1}^c \\ \pi_t^* \\ \pi_t^c \\ \pi_{t-1}^c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_1^* & \rho_2^* & 0 & \beta_1^* & \beta_2^* \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \alpha_1^* & \alpha_2^* & 0 & \phi_1^* & \phi_2^* \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_t^n \\ u_t^c \\ u_{t-1}^c \\ \pi_t^* \\ \pi_t^c \\ \pi_{t-1}^c \end{pmatrix} + \begin{pmatrix} v_{u,t} \\ e_{u,t} \\ 0 \\ v_{\pi,t} \\ e_{u,t} \\ 0 \end{pmatrix}$$

$$Q = E\left[\xi_t' \xi_t\right] = \begin{pmatrix} \sigma_{v_u}^2 & \sigma_{v_u,e_u} & 0 & \sigma_{v_u,v_\pi} & \sigma_{v_u,e_\pi} & 0 \\ \sigma_{e_u,v_u} & \sigma_{e_u}^2 & 0 & \sigma_{e_u,v_\pi} & \sigma_{e_u,e_\pi} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sigma_{v_\pi,v_u} & \sigma_{v_\pi,e_u} & 0 & \sigma_{v_\pi}^2 & \sigma_{v_\pi,e_\pi} & 0 \\ \sigma_{e_\pi,v_u} & \sigma_{e_\pi,e_u} & 0 & \sigma_{e_\pi,v_\pi} & \sigma_{e_\pi}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Using the state space model specification, Lee and Nelson estimates the stochastic trends from the observed unemployment rate and inflation rate, and simultaneously estimated the VAR parameters. There are few restrictions placed on the covariance of the disturbances. In their original specification, the covariance between the permanent and the cyclical disturbance terms of each related variable (i.e., σ_{v_u,e_u} and σ_{v_π,e_π}) are left unrestricted, which can have some bearing on the dynamics of the estimated system. Lee and Nelson implicitly restrict the cross-equation, cross-variable covariance terms to equal zero. In the Q matrix, the terms σ_{v_π,e_u} =0 and σ_{v_u,e_π} =0. That is, the covariance between, say, the disturbance to the cyclical inflation and the disturbance to the natural rate of unemployment and vice versa is restricted to be zero. This assumption seems reasonable and non-controversial.

$$Q^R = \begin{pmatrix} \sigma^2_{v_u} & \sigma_{v_u,e_u} & 0 & \sigma_{v_u,v_\pi} & 0 & 0 \\ \sigma_{e_u,v_u} & \sigma^2_{e_u} & 0 & 0 & \sigma_{e_u,e_\pi} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sigma_{v_\pi,v_u} & 0 & 0 & \sigma^2_{v_\pi} & \sigma_{v_\pi,e_\pi} & 0 \\ 0 & \sigma_{e_\pi,e_u} & 0 & \sigma_{e_\pi,v_\pi} & \sigma^2_{e_\pi} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Then, there are only 8 parameters to estimate in the Q matrix. With the covariance between the disturbances to the respective trend components left unrestricted, a non-zero covariance estimate would suggest long run dynamics that may be hard to reconcile with long-run neutrality of nominal shocks. On the other hand, if it is positive, it could reflect the costs in unemployment terms of a higher trend inflation rate.

VIII ROBUSTNESS OF THE SHORT-RUN PHILLIPS CURVE

Tables 4 and 5 display the parameter estimates from Lee and Nelson baseline model alongside the estimates from the higher likelihood case. Looking first at Table 1, it is notable that the error term associated with the long-run trend of unemployment (the natural rate) has a substantially larger estimated value in the alternative estimation. In contrast, the alternative estimate displays an estimated standard error in the underlying trend inflation rate equation that is only about 1/3 the estimated value in the Lee and Nelson baseline. So the alternative estimation shows more variability in the trend unemployment rate error, and less variability in the trend inflation rate error.

The graphics in Figures 1 and 2 show the unemployment time-series and the inflation time-series respectively. In Figure 13, the chart overlays the estimate of the natural rate from the Lee and Nelson baseline as well as the estimated natural rate from the alternative findings. There is notable contrast between the slow moving and less variable natural rate estimate from Lee and Nelson baseline (NRU_LN) versus the more volatile and jagged movements in the natural rate estimate from the alternative (NRU_ALT).[16] The graphical contrast reflects the difference in the estimated standard error in the natural rate equation disturbance term. The alternative estimate indicates greater variability in the standard error of the disturbance term, so the graphical display of a time-series estimate for the natural rate that is more variable than the Lee and Nelson baseline estimate is not surprising.

Figure 14 displays the CPI inflation rate along with the two time-series estimates for the trend inflation rate from the Lee and Nelson baseline and the alternative parameter set, respectively. The key distinction between the two trend inflation estimates arises during the mid-1970s. The Lee and Nelson baseline estimate suggests that during the 1970s inflation rose because of an increase in trend inflation, whereas the alternative estimate suggests that the 1970s inflation was largely a temporary phenomenon. Similar to the discussion above, the smoother path for the inflation trend estimate from the alternative estimate is consistent with the finding that the standard error of the disturbance term in that equation is much smaller than in the Lee and Nelson baseline. In the alternative estimate, more explanation of the movements in CPI inflation shift to the cyclical disturbance term.

Figures 15 and 16 display the cyclical unemployment estimates versus the cyclical inflation estimates for the Lee and Nelson baseline and the alternative, respectively. In Figure 15, both series display striking negative correlation throughout

---

16 The fluctuating natural rate estimate from the alternative is similar to the movements in the natural rate estimate proposed by King and Morley (2007), who use a multi-variate Beveridge-Nelson decomposition to estimate the natural rate.

the sample, with particularly sharp movements in the 1974-78 and 1981-1984 periods. There appears to be less striking correlation in Figure 16. The negative correlation is striking from 1973-75 as well as from 1981-1984, and there appears to be notable negative correlation between cyclical inflation and cyclical unemployment from 1966 through 1986. From 1987 onward, there appears to be a negative correlation, but its amplitude is modest.

Figure 17 compares the cyclical unemployment measures of the Lee and Nelson baseline with those from the alternative estimate, and Figure 18 compares the respective cyclical inflation estimates. For the cyclical unemployment rates, the time-series movements differ most notably in the mid-1970s and early 1980s. For the alternative, the cyclical unemployment rate moves negative sharply in 1973 and 1974. In contrast, the Lee and Nelson baseline spikes upward and becomes a large positive in 1975 and 1976. From the perspective of the alternative, the 1974 increase in the inflation rate was largely a temporary phenomenon, and the sharp negative movement of the cyclical unemployment rate is consistent with this decomposition. That is, the alternate estimate of the natural rate rose in 1974 by more than the unemployment rate rose. Figure 13 illustrates this point clearly.

Returning to Table 4, the covariance of the trend inflation rate error and the cyclical inflation rate error is negative in the alternative estimation, whereas it is positive in the Lee and Nelson baseline. There appears to be no substantial co-variation between the long-term error terms of unemployment and inflation in either model estimate, although in the alternative estimate, the covariance is positive and notably larger than the estimate in Lee and Nelson. Further, the co-variation between the reduced-form error terms in the respective cyclical equations is notably weaker in the alternative estimate, and shifts to a positive value when the zero restrictions on parameters $\beta_1^*, \beta_2^*, \phi_1^*$ and ϕ_2^* are imposed. With respect to the Phillips Curve implications, the first column of unrestricted estimates for the alternative estimation shows parameter estimates for α_1^* and α_2^* that are smaller than the Lee and Nelson baseline estimates and that are not significantly different from zero. In the restricted model case, that is, when the parameters $\beta_1^*, \beta_2^*, \phi_1^*$ and ϕ_2^* are set to zero, at least α_1^* appears to be significantly different from zero in the alternative estimate. However, in the alternative estimation, the restricted estimation generates a log-likelihood value of 92.917. In this case, the restrictions are rejected at the 5 % significance level.

Additional evidence in Table 5, which displays parameter estimates from the structural VAR, suggests that the sum of coefficients for contemporaneous and lagged cyclical unemployment measures in the cyclical inflation equation is a bit more than half the value found in the Lee and Nelson baseline. Further, the estimate of the sum of the α_i coefficients is statistically significant only in the restricted estimate, which is rejected using the likelihood ratio test at a 5 percent confidence level.

Data for CPI inflation and the unemployment rate extend through 2008Q1, so Tables 6 and 7 display estimation results that exploit these additional observations. The first and third columns list the parameter estimates from starting from the same initial values as used in the Lee and Nelson baseline. The second and fourth columns list the parameter estimates one gets from starting from the alternative initial values. First, as in the discussion above, the alternative estimation shifts variability into the permanent disturbance associated with the natural rate of unemployment, and shifts some variability away from the inflation trend disturbance and toward the cyclical inflation disturbance.

In Tables 6 and 7, the coefficient estimates related to the Phillips Curve correlation are smaller in the alternative than in the estimate similar to the Lee and Nelson baseline. In contrast to the results above, the restriction that the parameters $\beta_1^*, \beta_2^*, \phi_1^*$ and ϕ_2^* are zero is not rejected at the 5 percent critical value (Likelihood Ratio statistics of 6.6 and 3.3 respectively, critical value χ^2 (.05, 4) =9.48). Finally, the Phillips curve parameters in the restricted case are statistically significant only in the alternative estimation. The results in support of the Phillips Curve correlation in the alternative parameter set appears a curious finding (see the graphical displays described in Appendix A) and will be investigated further in future research.

VIII CONCLUSIONS

This paper employs simple empirical models to investigate the empirical Phillips Curve in US data. The research follows on the work and findings in King and Watson (1994) and examines how their evidence is altered by the addition of over 15 years of data observations.

The separate approach to the empirical Phillips Curve evidence forms estimates of the natural rate of unemployment in order to isolate an observable Phillips Curve in US data. King and Morley (2007) specify a structural vector auto-regression model from which a natural rate of unemployment estimate is easily derived. Findings in this paper show that main conclusions in King and Morley (2007) hold when the model is estimated with additional data observations. There appears a notable short-term Phillips

Curve in the US data when the full-sample multivariate Beveridge-Nelson decomposition produces the natural rate of unemployment estimate. These results hold also when the full-sample structural VAR generates the natural rate of unemployment estimate.

This paper assesses an alternative estimate of the natural rate of unemployment to uncover the Phillips Curve relationship in the data. Two features of this approach appear worthy of further investigation. The results in King and Morley indicate a robust Phillips Curve relationship. It may be worth investigating the dependence of those results on the identification restrictions that are implicit in the model that generates the estimate for the natural rate of unemployment. Results suggest that a three-variable VAR model using monthly data can generate results similar to those results in the King and Morley investigation as long as the model specification employs the inflation rate in levels. When the inflation rate is first differenced, the relationship with inflation appears weaker, although the movements in the related cyclical unemployment estimates are correlated with a band pass filter focusing on frequencies between two and eight years, the business cycle frequencies. This latter finding will be the subject of further investigation.

This paper also re-examines the model specification in Lee and Nelson (2007) and, while using the same data set, finds a set of parameter estimates for which the log likelihood value is higher than the baseline model in Lee and Nelson. These new results offer weaker support for the relevance of the Phillips Curve correlation in U.S. data. Foremost, the results suggest a notably weaker contemporaneous covariance between the error terms associated with the cyclical measures of inflation and unemployment. Also, in the cyclical inflation equation, lags of the cyclical unemployment measure are associated with coefficient values that are smaller and that have only marginal significance. Finally, the contemporaneous covariance between the error terms in the long-term trend equations for both unemployment and inflation becomes positive. There is no long-term trade off of trend inflation for trend unemployment; rather, there may be a positive long-run correlation between them, consistent with intuitions that suggest either that higher trend inflation has associated costs that give rise to a higher natural rate of unemployment, or that real technology shocks that increase the natural rate of unemployment, reduces the growth rate of real activity, and leads to higher trend inflation. However, this result conflicts with standard assumption of long-run neutrality. Further work will assess the sensitivity of the findings to imposing long-run neutrality as a restriction. Using an extended data sample that includes nearly six years of additional observations, the restricted model estimate is not rejected and the coefficients associated with the Phillips Curve appear negative and significant.

From a more technically descriptive point of view, the new results achieve a higher likelihood value by pushing more volatility into the natural rate of unemployment time-series estimate. The additional volatility of the natural rate of unemployment estimate provides a more variable time-series estimate, which may generate objections by its inherent volatility. The more stable natural rate estimate in the original Lee and Nelson results accords more with accepted beliefs about the time-series volatility of natural rate estimates. In contrast, King and Morley (2007), who employ a multi-variate Beveridge-Nelson decomposition to isolate a natural rate of unemployment, generate a natural rate estimate that displays time-series properties that are similar to those in the new estimates of the Lee and Nelson model.

APPENDIX A

Figures A1 and A2 display the (alternative) time-series estimates for the natural rate and the trend inflation rate, denoted as cyclicalu_ll123 and cyclicalinf_ll123, respectively, estimated using the extended sample. The charts also display the two estimates (Lee and Nelson and the alternative) for the trend series estimated over the original sample in Lee and Nelson (2007). The time-series for both the natural rate of unemployment and the trend inflation rate estimated using the extended sample virtually duplicates the relevant time-series from the alternative estimates, denoted cyclicalu_alt and cyclicalinf_alt, respectively. Conversely, Figures A3 and A4 display time-series estimates for the natural rate and the trend inflation rate, denoted as cyclicalu_ll149 and cyclicalinf_ll149, that mimic the relevant estimate from the Lee and Nelson baseline. Using the full sample of data, the estimates depend crucially on the initial parameter values set before employing the Kalman filter to estimate the model by maximum likelihood.

REFERENCES

Arino, Miguel A. and Paul Newbold. (1998) "Computation of the Beveridge-Nelson Decomposition for Multivariate Economic Time-series." *Economics Letters*. 61: 37-42.

Beveridge, Stephen and Charles R. Nelson. (1981) "A New Approach to Decomposition of Economic Time-series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle." *Journal of Monetary Economics*. 7, 151-174.

Blanchard, Olivier and Danny Quah. (1989) "Dynamic Effects of Aggregate Demand and Supply Disturbances." American Economic Review (September) 79 (4): 655-673.

Christiano, Lawrence J. and Terry J. Fitzgerald (2003) "The Business Cycle: It's Still a Puzzle," Federal Reserve Bank of Chicago Economic Perspectives 56-83.

Friedman, Milton (1968) "The Role of Monetary Policy." *American Economic Review* (March) 58 (1), 1-17.

Harvey, Andrew (2007) "Modeling the Phillips Curve with Unobserved Components," Manuscript, Cambridge University.

King, Robert G. and Mark W. Watson (1994) "The Post-War US Phillips Curve: A revisionist econometric history," *Carnegie-Rochester Conference Series on Public Policy* 41.

King, Thomas B. and James Morley. (2007) "In Search of the Natural Rate of Unemployment," *Journal of Monetary Economics*. May, 54: 550-564.

Kuttner, Kenneth N. (1994) "Estimating Potential Output as a Latent Variable," Journal of Business and Economic Statistics, (July) 12, 3:361-368.

Lee, Jaejoon and Charles R. Nelson (2007) "Expectation Horizon and the Phillips Curve: The Solution to an Empirical Puzzle," *Journal of Applied Econometrics*, 22: 161-178.

Phelps, Edmund (1967) "Phillips Curves, Expectations of Inflation, and Optimal Inflation over Time, Economica 135: 254-281.

Salemi, Michael K. (1999) "Estimating the Natural Rate of Unemployment and Testing the Natural Rate Hypothesis." *Journal of Applied Econometrics* 14, 1-25.

Staiger, Douglas, James Stock, and Mark Watson. (1997) "NAIRU, Unemployment, and Monetary Policy" *Journal of Economic Perspectives*. (Winter) 11, 1: 828-864.

Figure A1: Natural Rate of Unemployment Estimates

Source: Bureau of Labor Statistics,

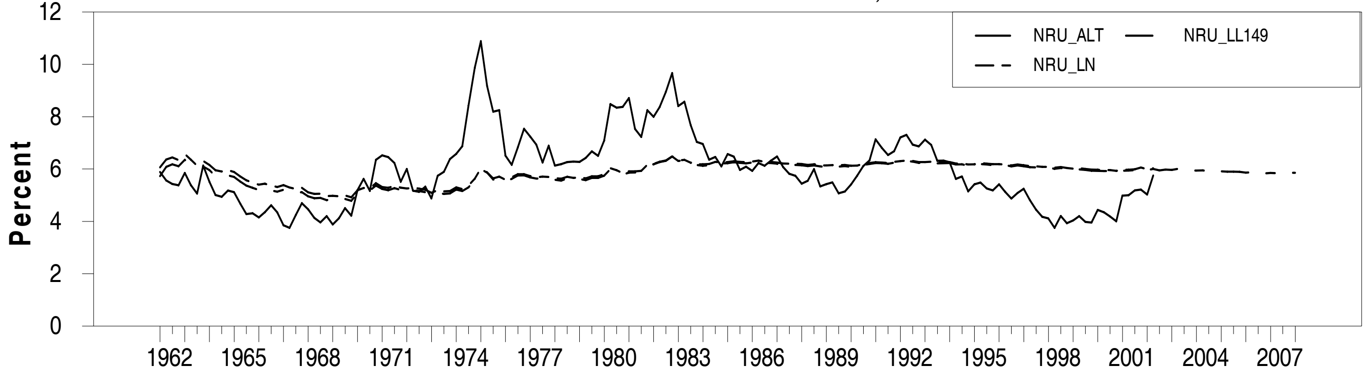


Figure A2: Trend Inflation Rate Estimates

Source: Bureau of Labor Statistics,

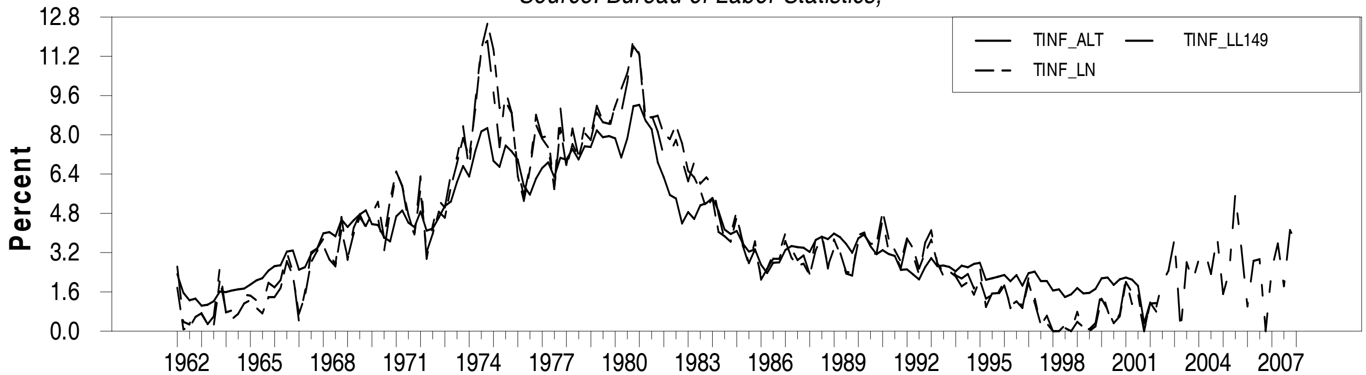


Figure A3: Natural Rate of Unemployment Estimates

Source: Bureau of Labor Statistics,

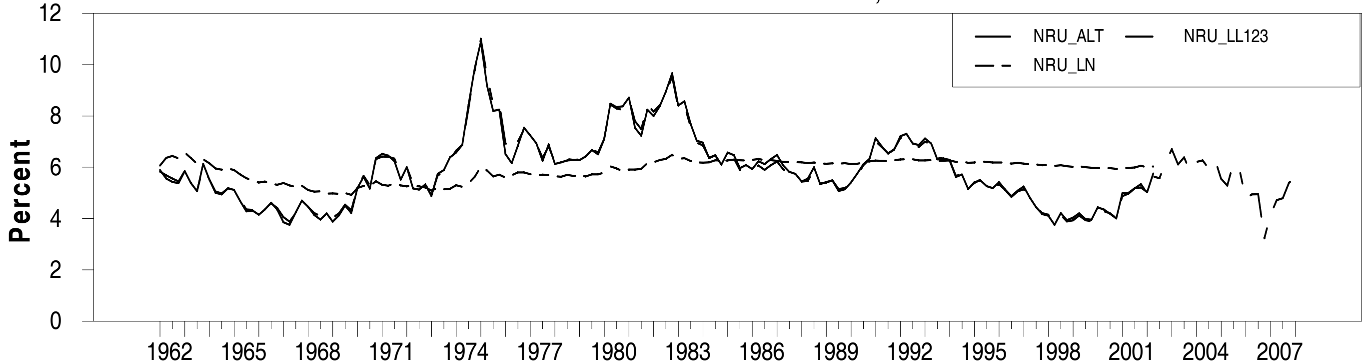


Figure A4: Trend Inflation Rate Estimates

Source: Bureau of Labor Statistics,

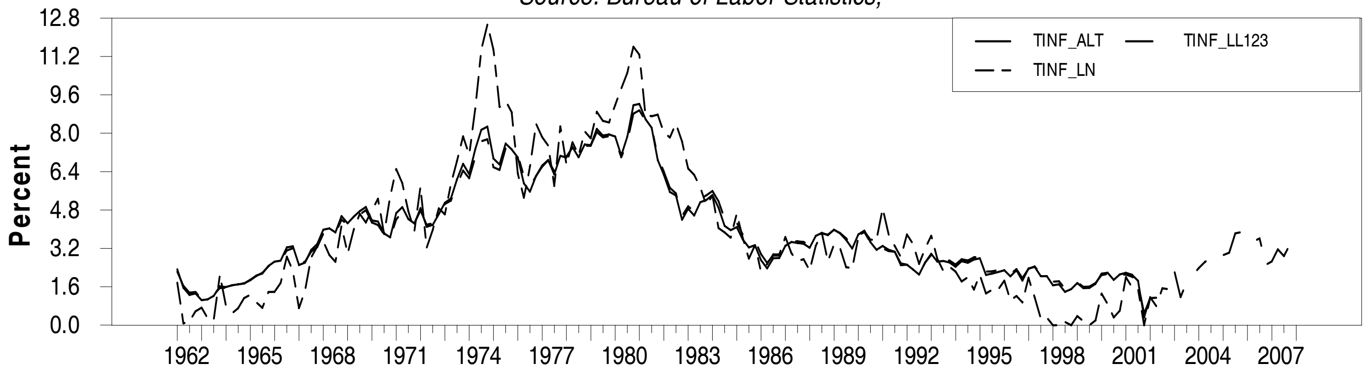


Chart 1: Quarterly Model Data in Level and Difference Form

Sample: 1950:2 2007:1

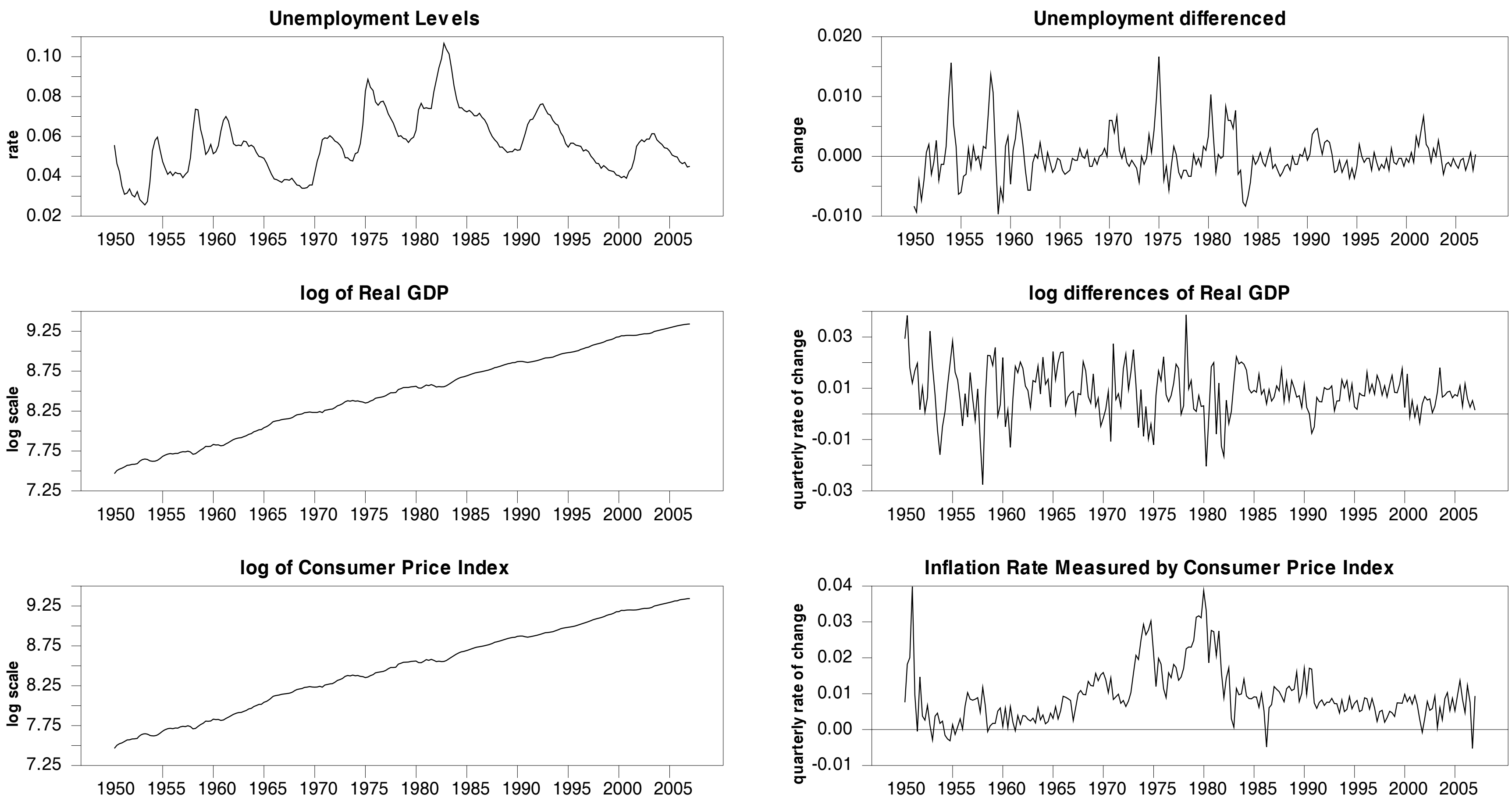


Chart 1a: Quarterly Inflation Data in Difference Form

Sample: 1948:3 2007:1

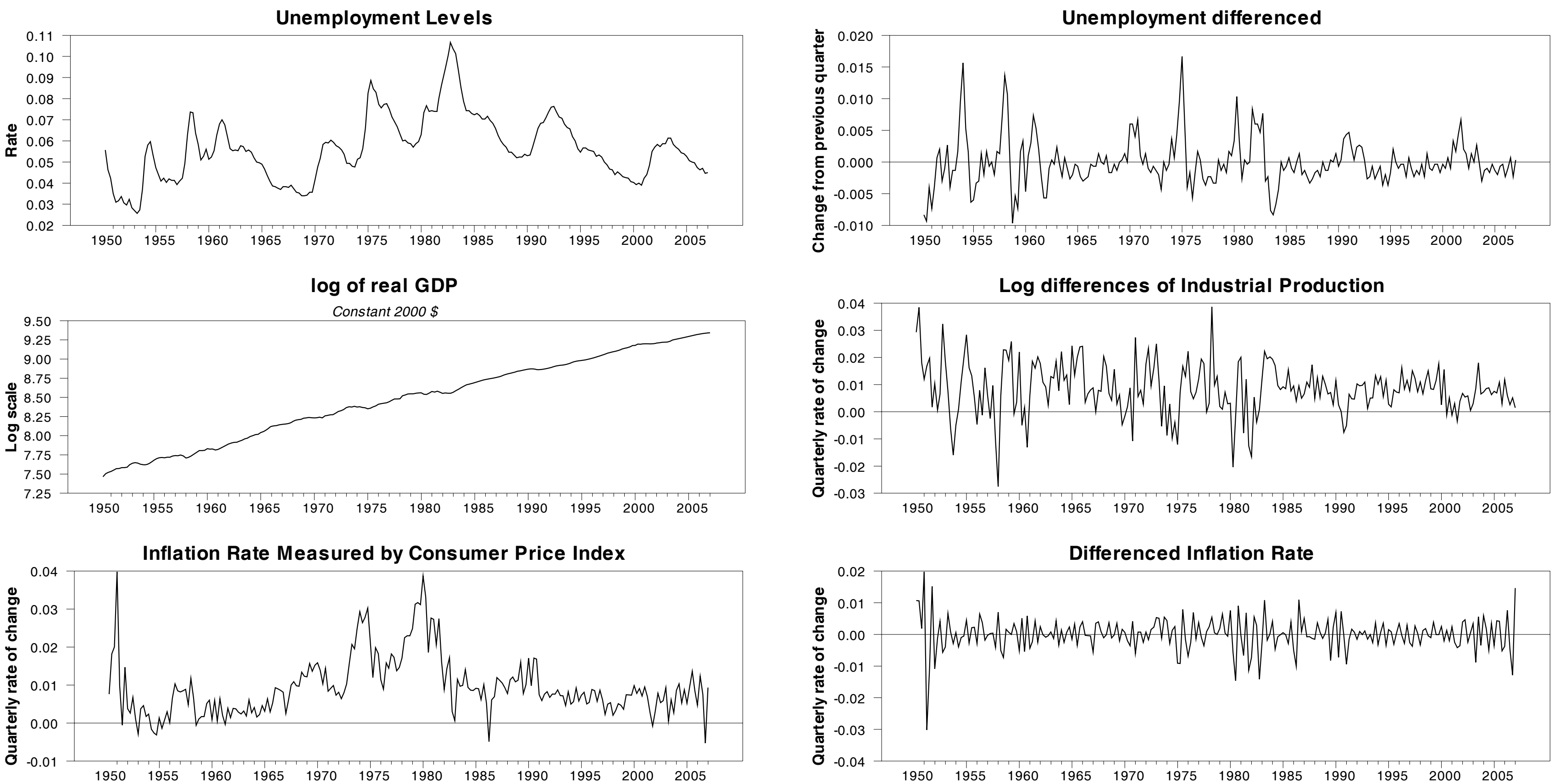


Chart 2: Monthly Model Data in Level and Difference Form

Sample: 1948:2 2007:12

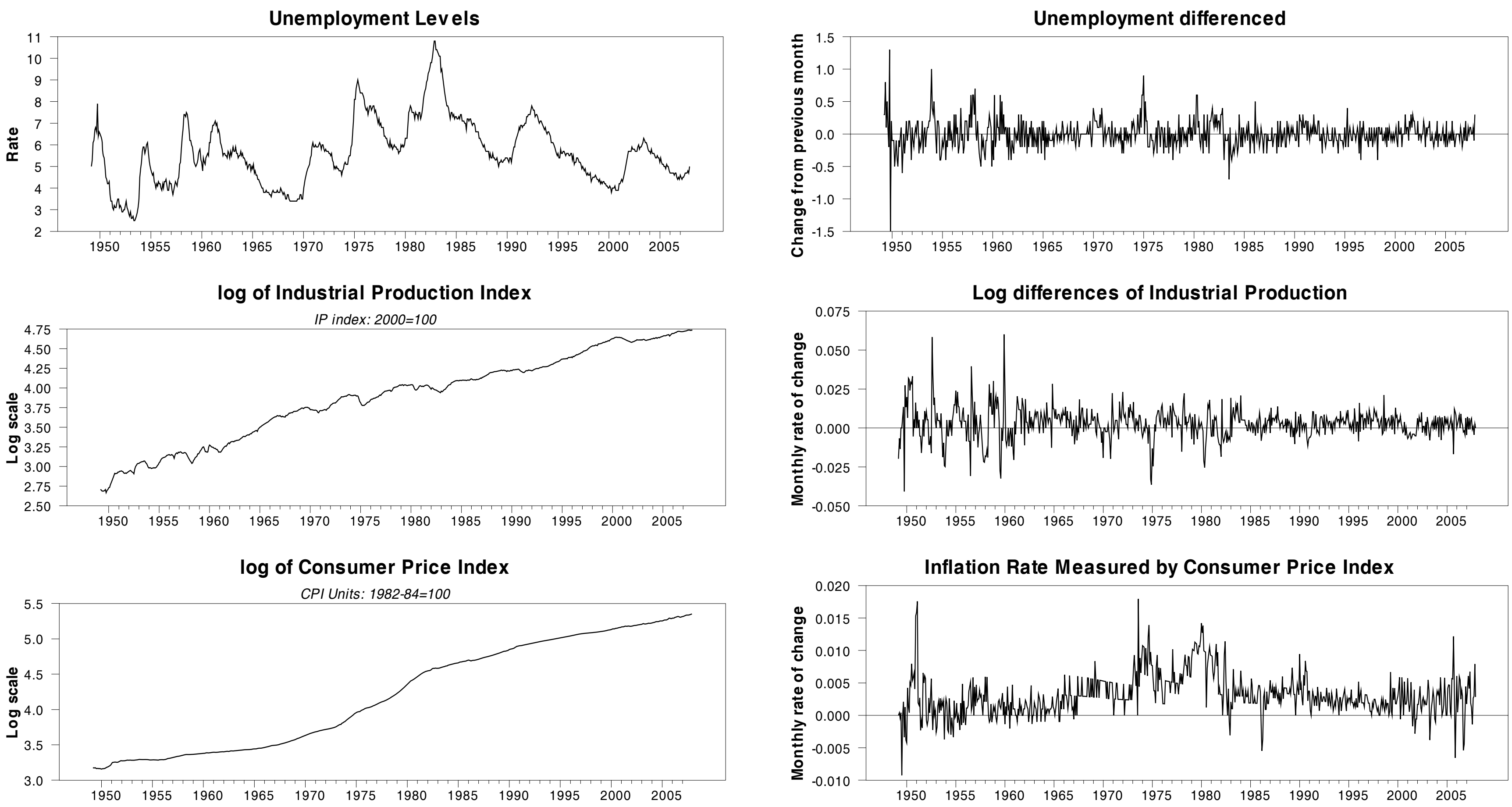
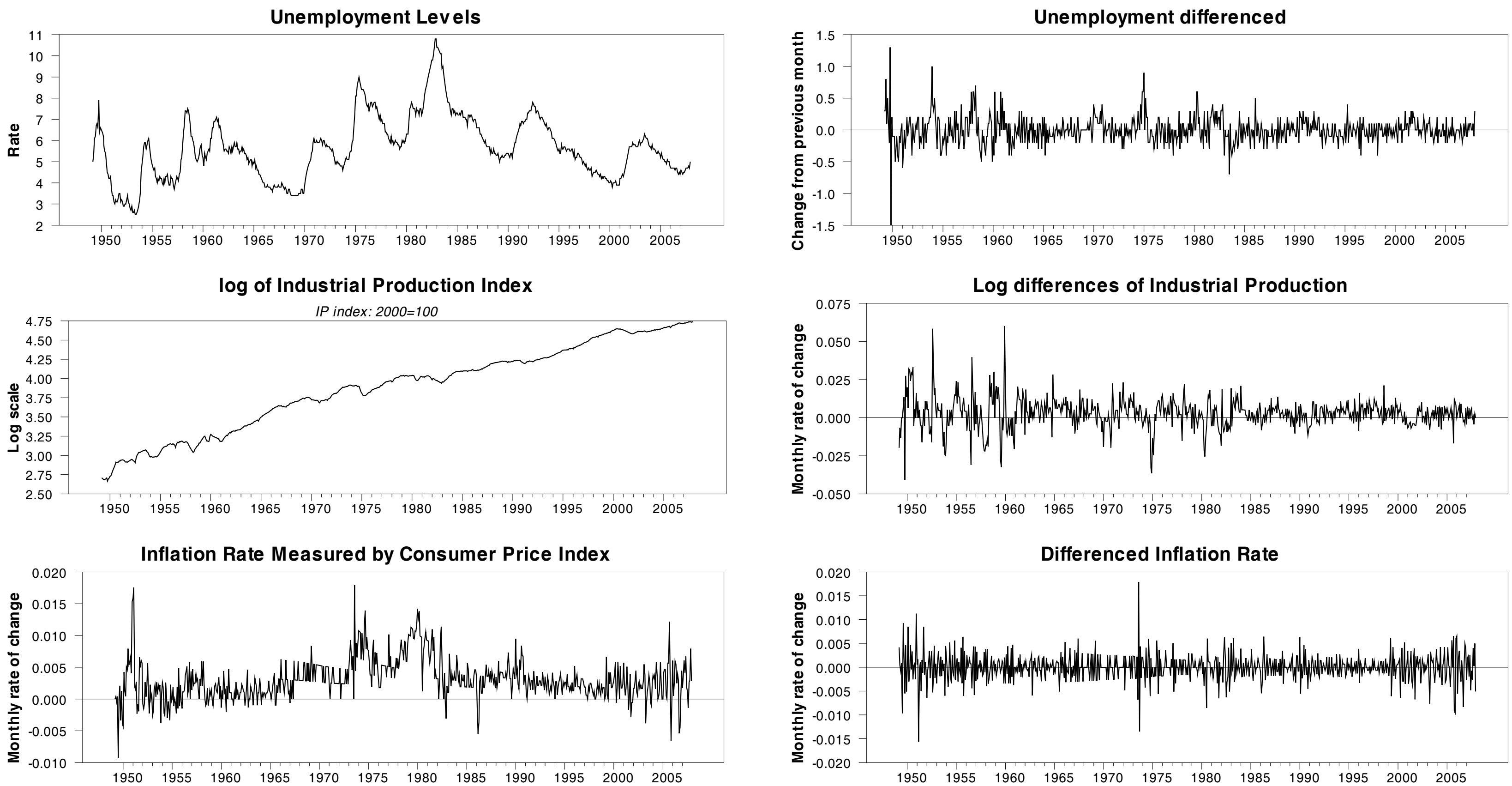


Chart 2a: Monthly Inflation Data in Difference Form

Sample: 1948:3 2007:12



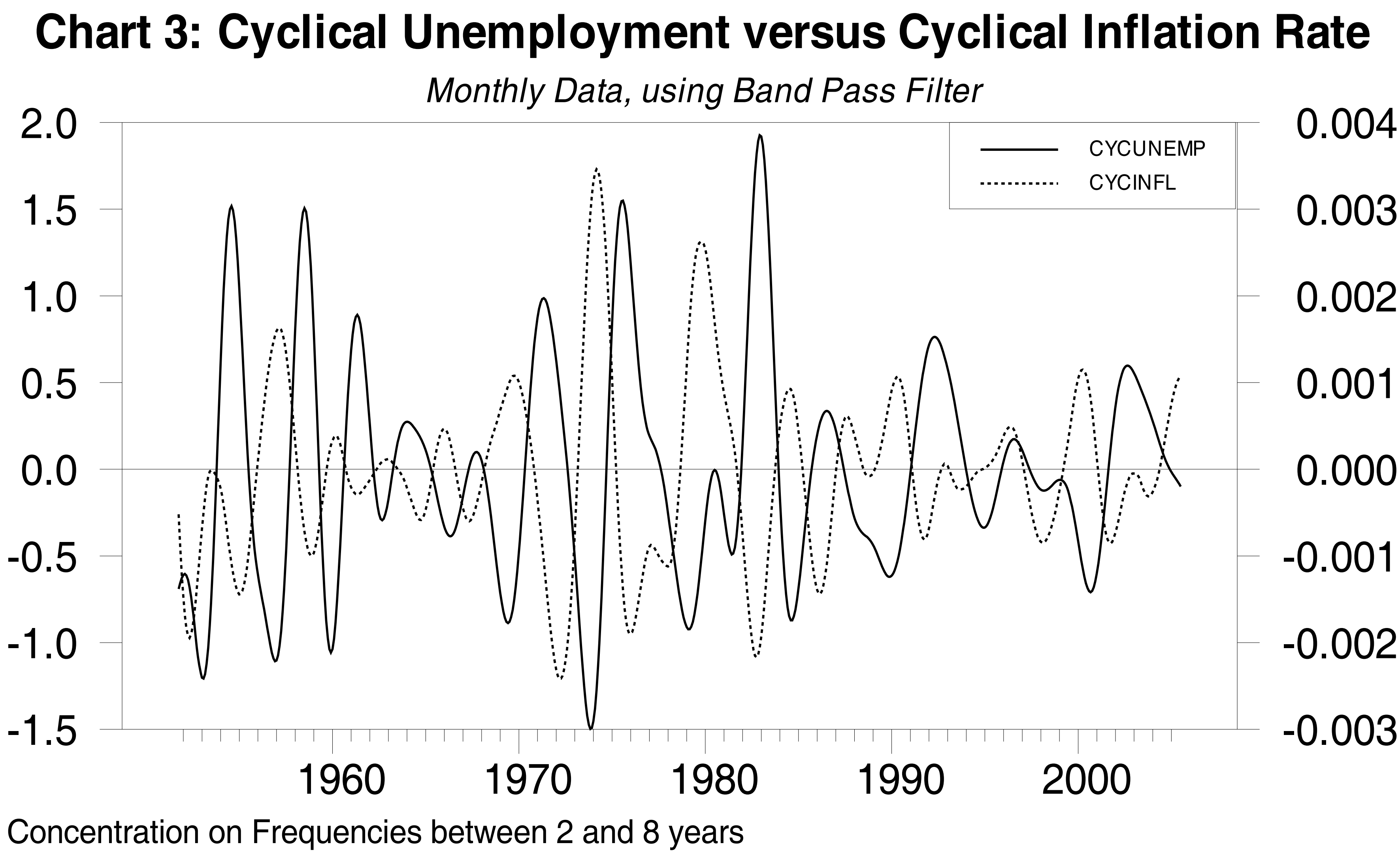


Chart 3: Cyclical Unemployment versus Cyclical Inflation Rate
Monthly Data, using Band Pass Filter
CYCUNEMP
CYCINFL
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
0.004
0.003
0.002
0.001
0.000
-0.001
-0.002
-0.003
1960
1970
1980
1990
2000
Concentration on Frequencies between 2 and 8 years

Chart 4 Vector Auto-regression Using Inflation Levels

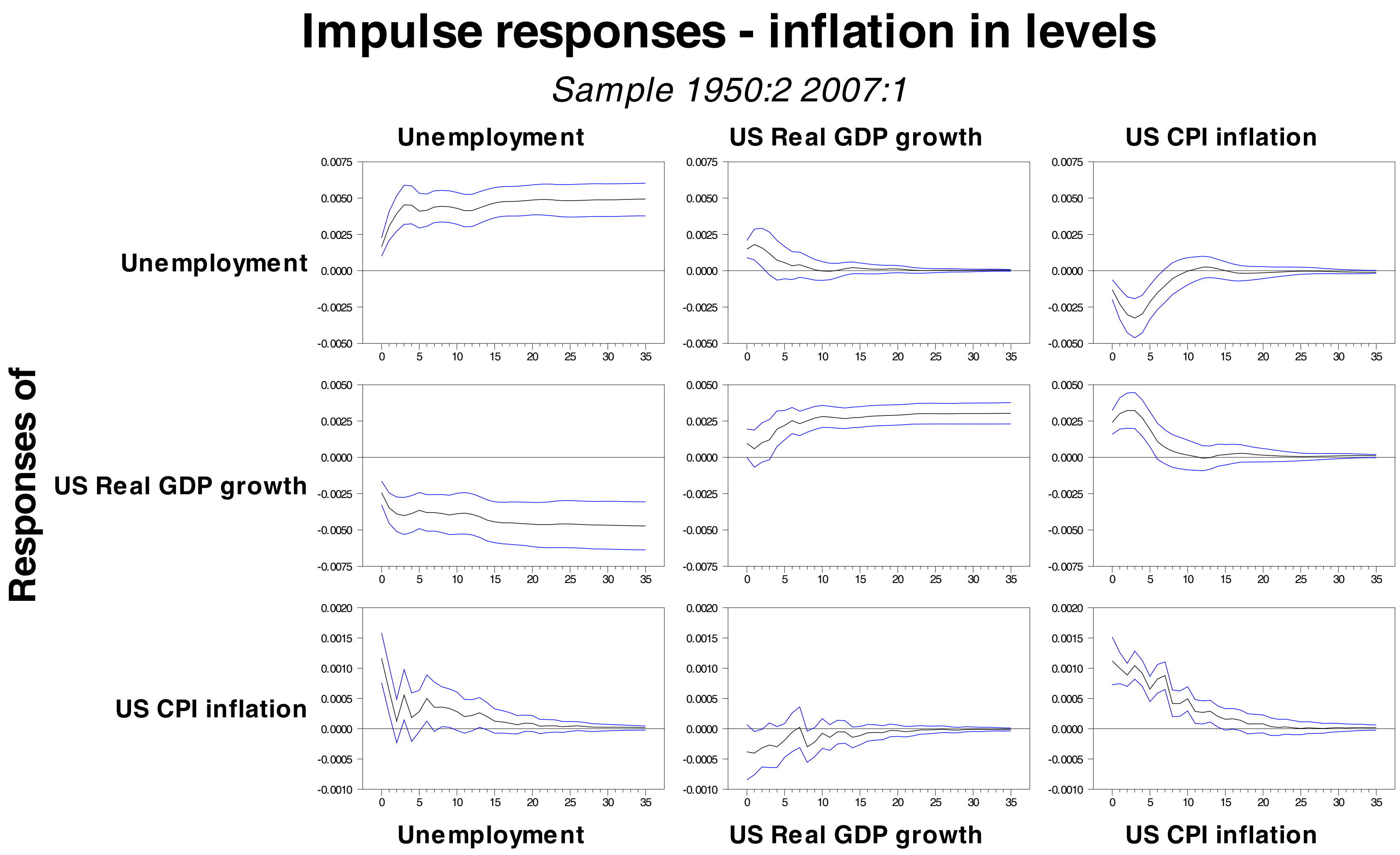


Chart 5 Vector Auto-regression – further results

Impulse responses - inflation in first differences

Sample - 1950:2 2007:1

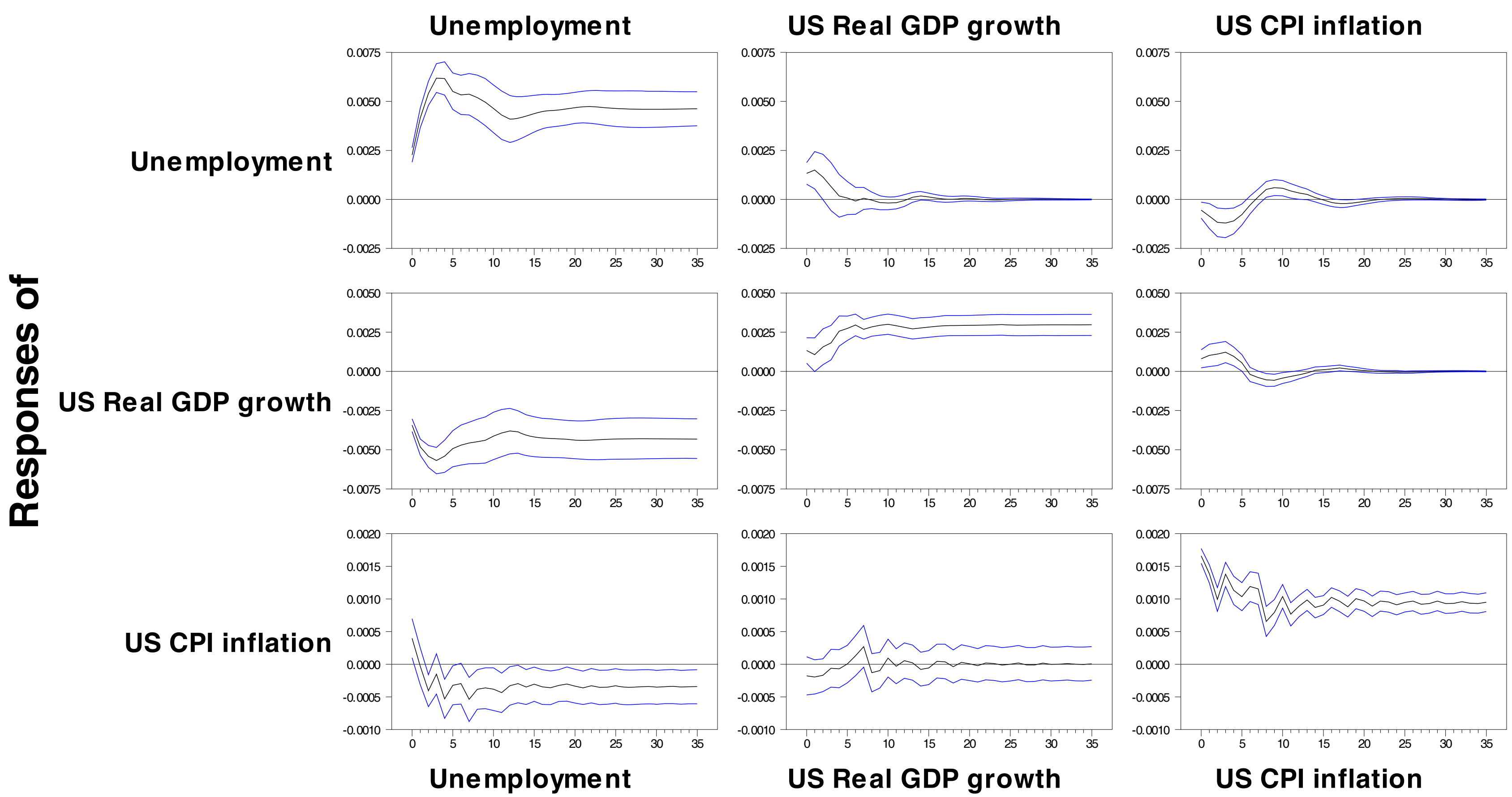


Chart 7: Unemployment Versus Natural Rate Estimates

Estimate from Accumulation of Permanent Shocks from Structural VAR

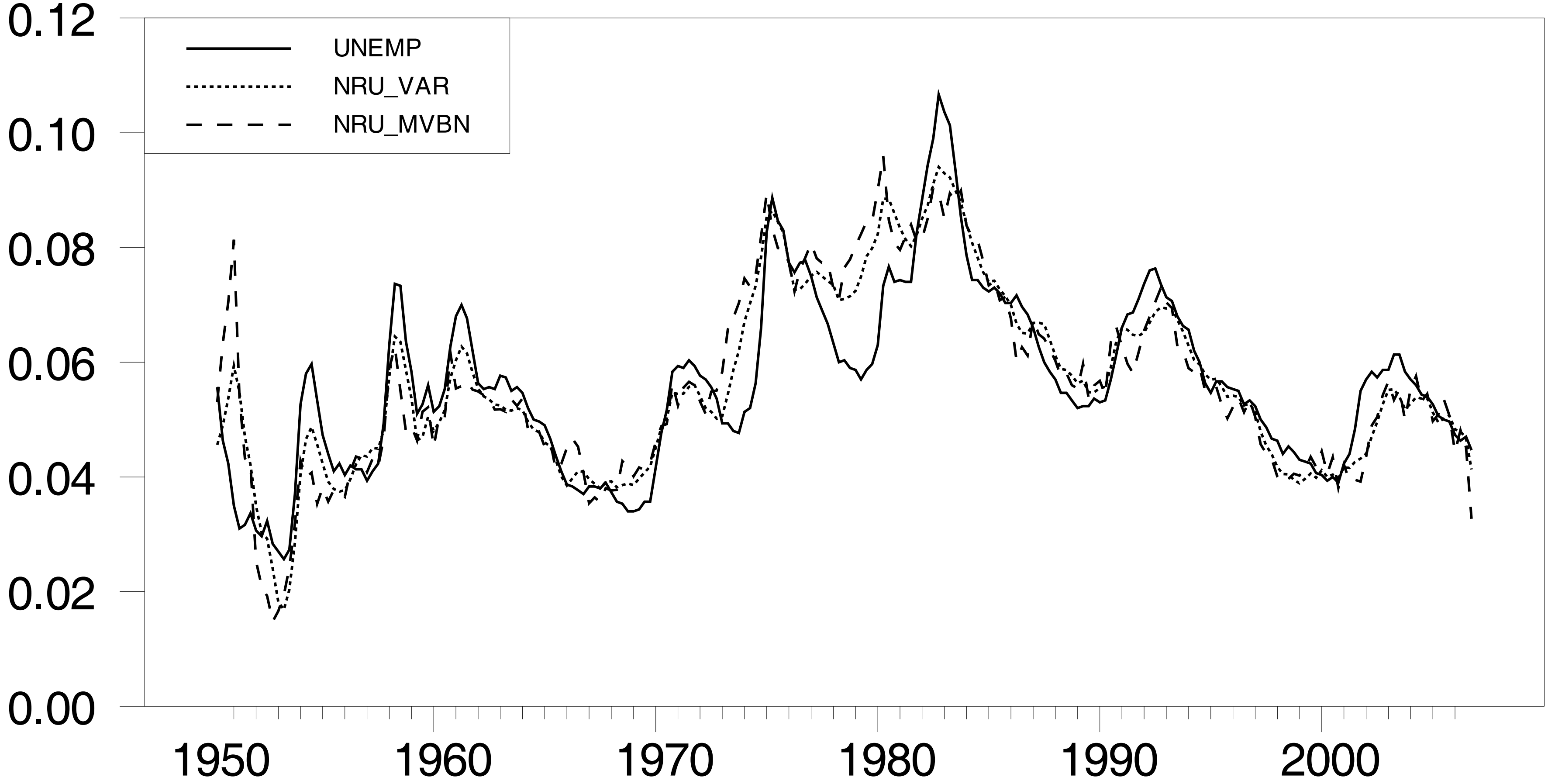


Chart 8: Cyclical Unemployment Estimates versus Inflation

derived from MVBN and VAR based Natural Rate Estimates

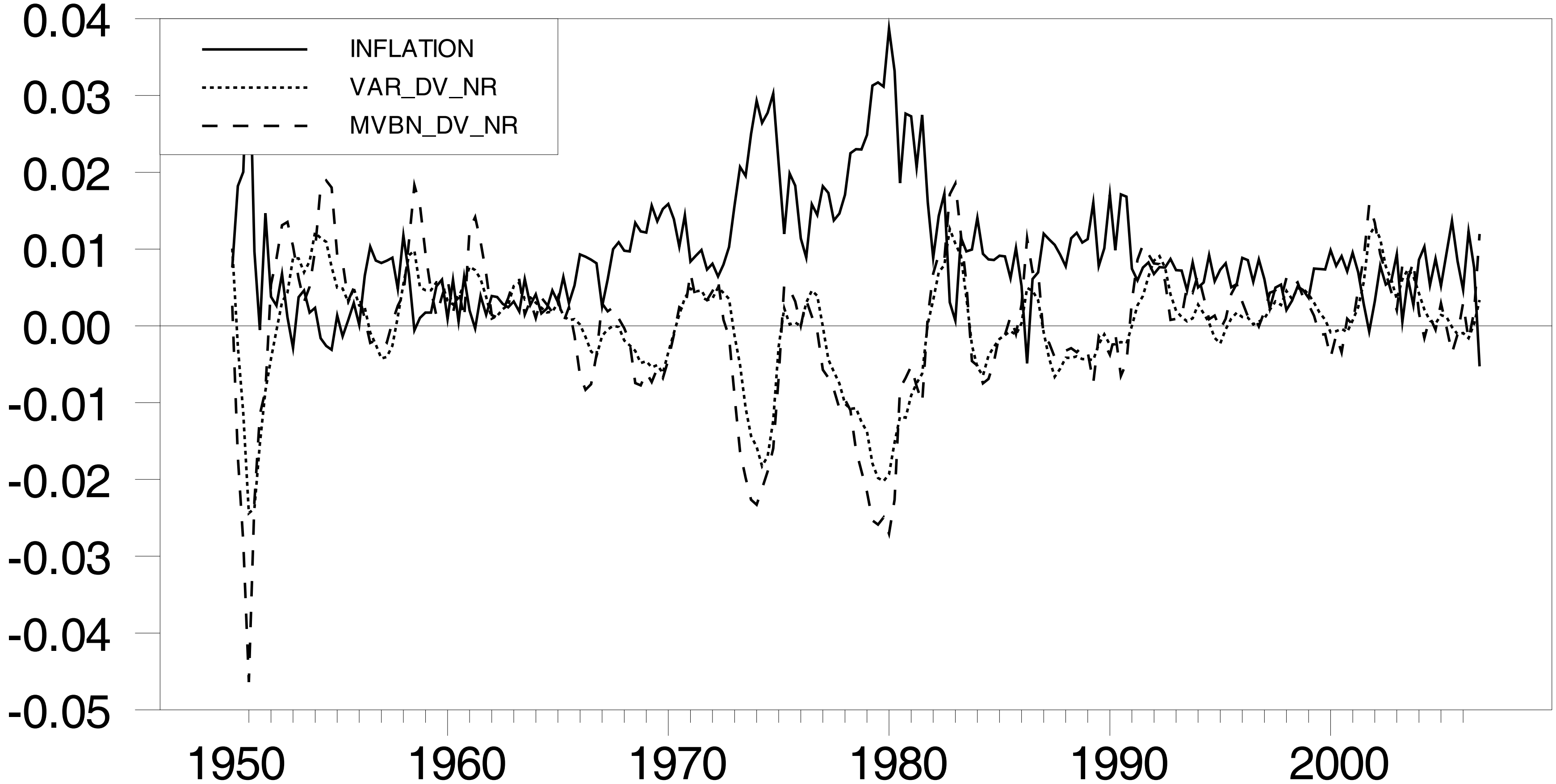
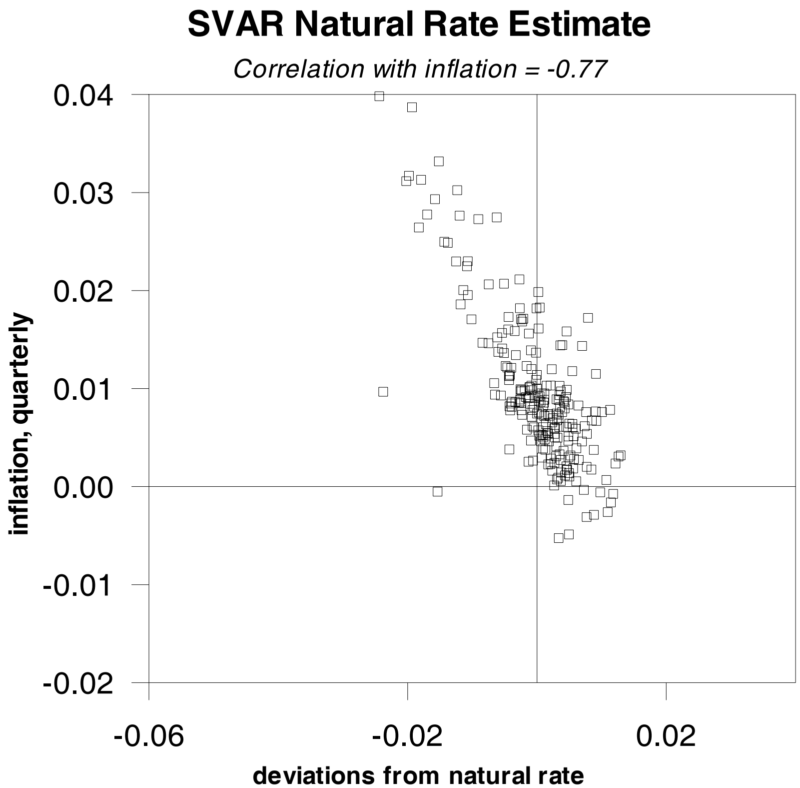


Chart 9: Phillips Curve: Cyclical Unemployment versus Inflation

Using 1950:2 2007:1 Full-Sample



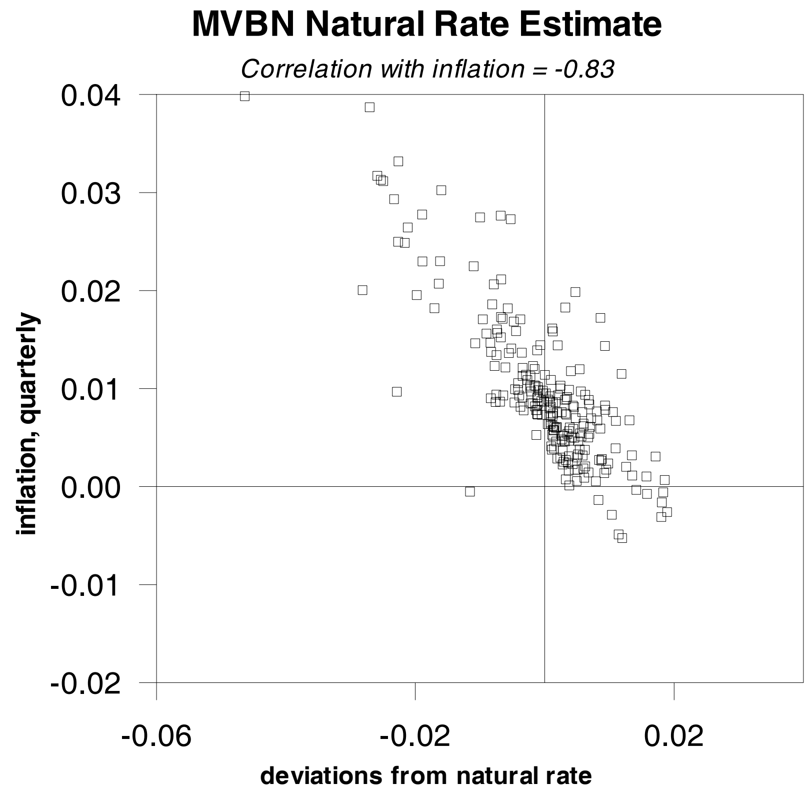


Chart 10: VAR Model Using Monthly Data Series

Impulse responses - Inflation in Levels

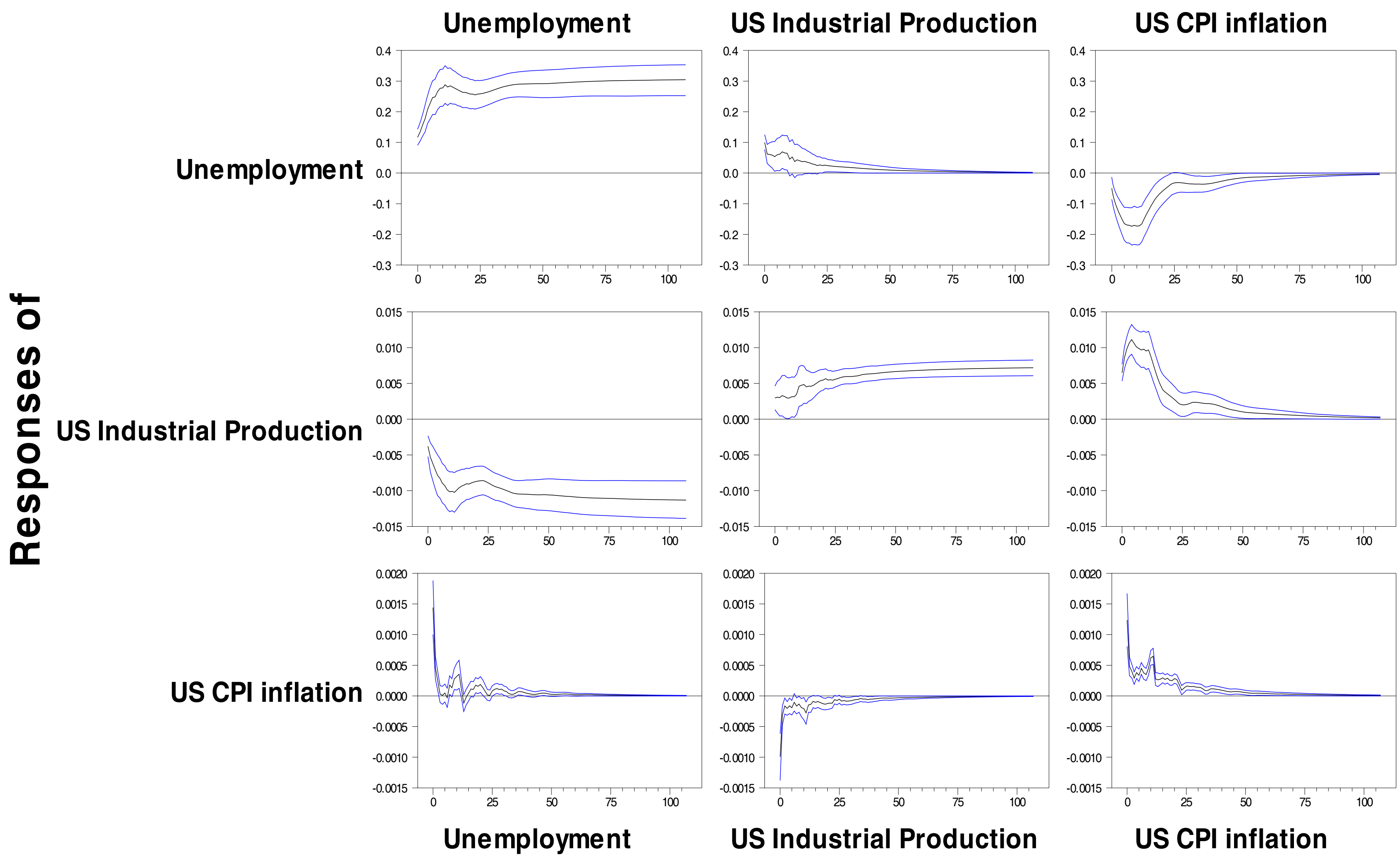


Chart 11: Inflation versus Cyclical Unemployment Estimates

derived from MVBN and SVAR Natural Rate Estimates, Monthly

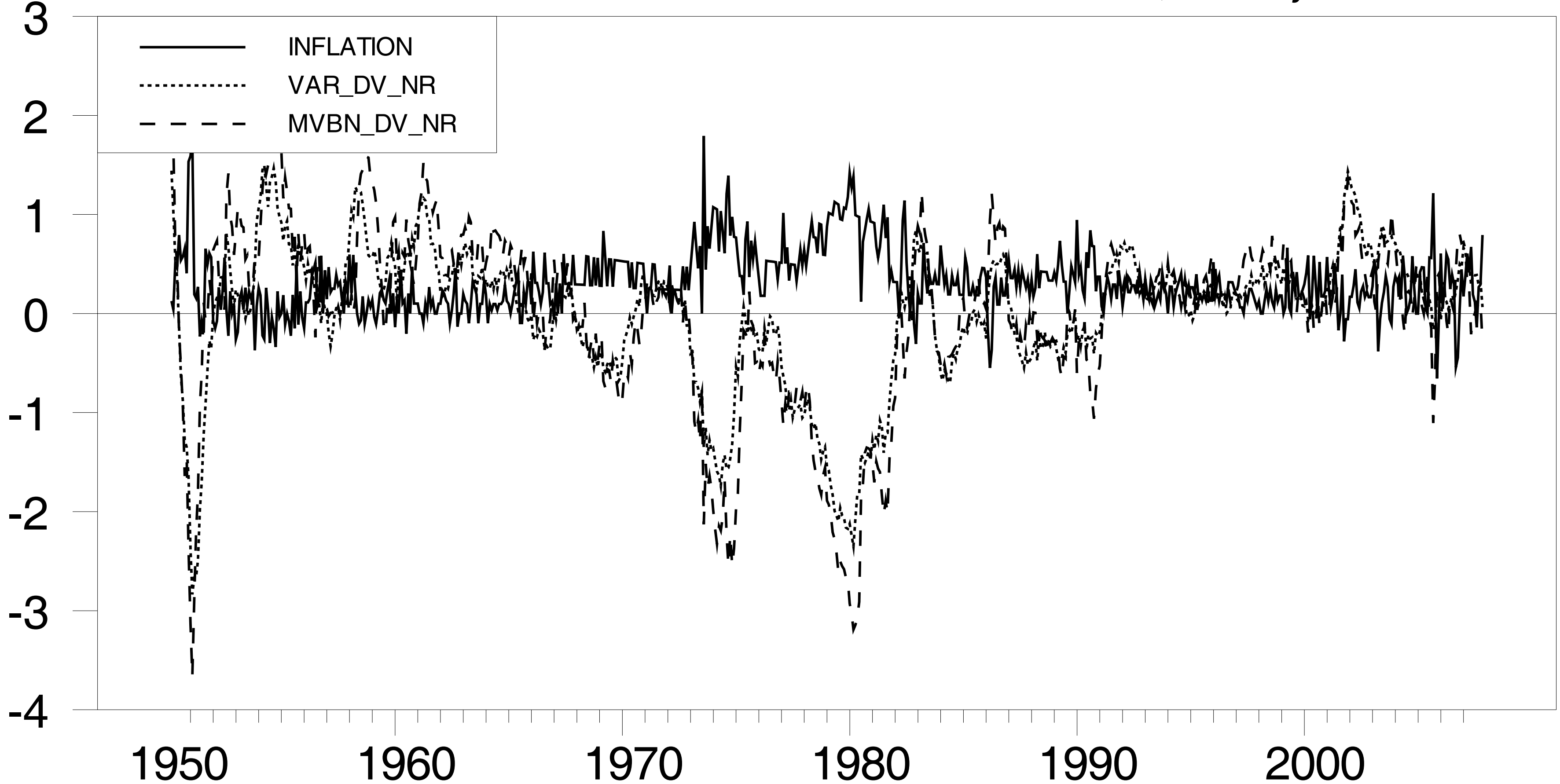
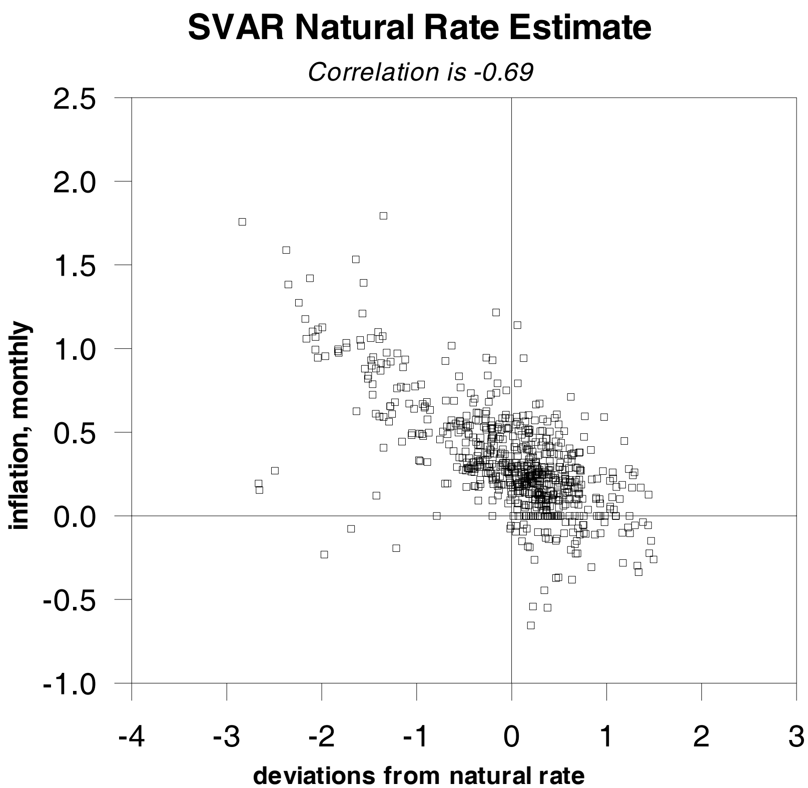


Chart 12: Phillips Curve: Cyclical Unemployment versus Inflation

Using 1949:2 2007:12 Full-Sample, Monthly Data



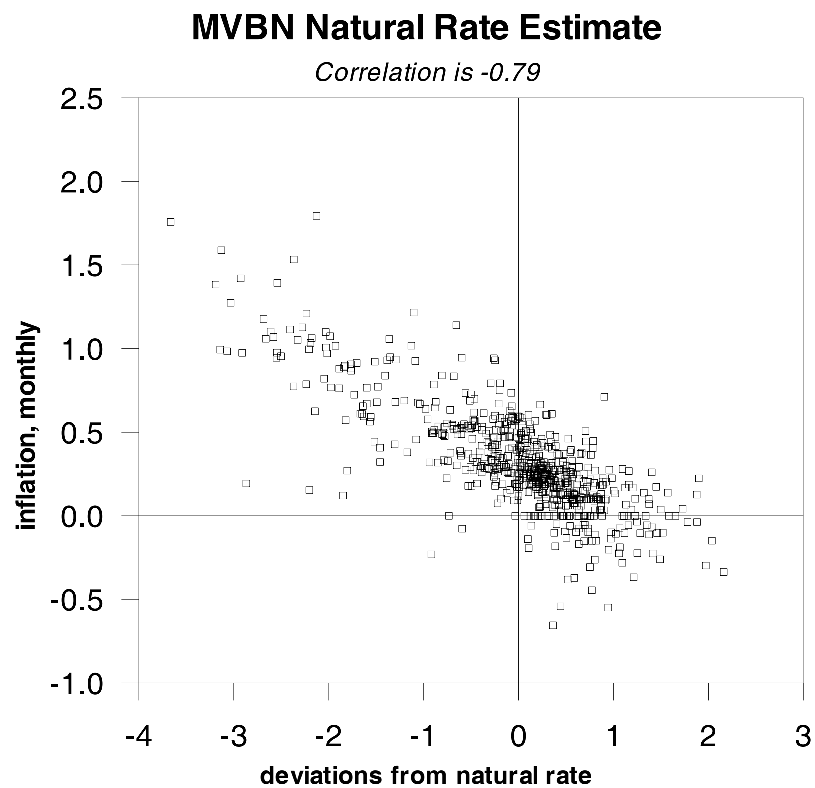


Figure 13: Unemployment Rate Versus Estimates of Natural Rate

Source: Bureau of Labor Statistics,

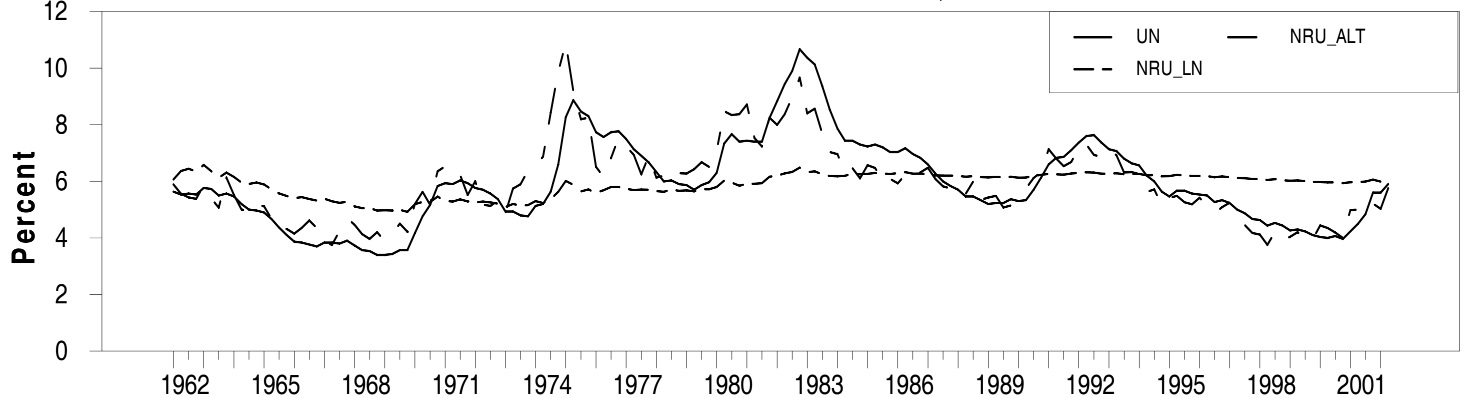


Figure 14: CPI Inflation Rate Versus Estimates of Trend Inflation Rate

Source: Bureau of Labor Statistics,

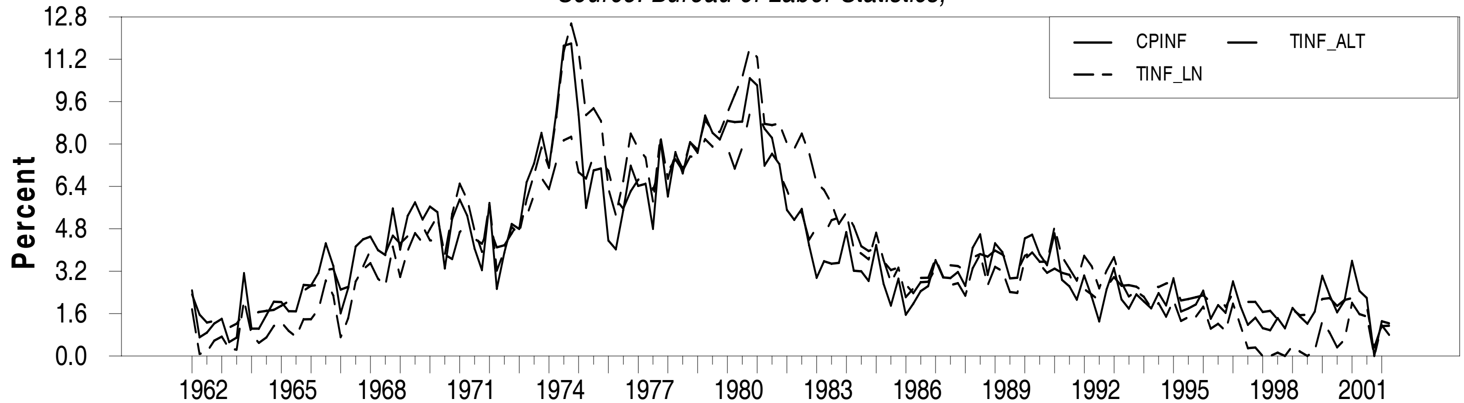


Figure 15: Cyclical Estimates in Lee and Nelson

Source: Bureau of Labor Statistics,

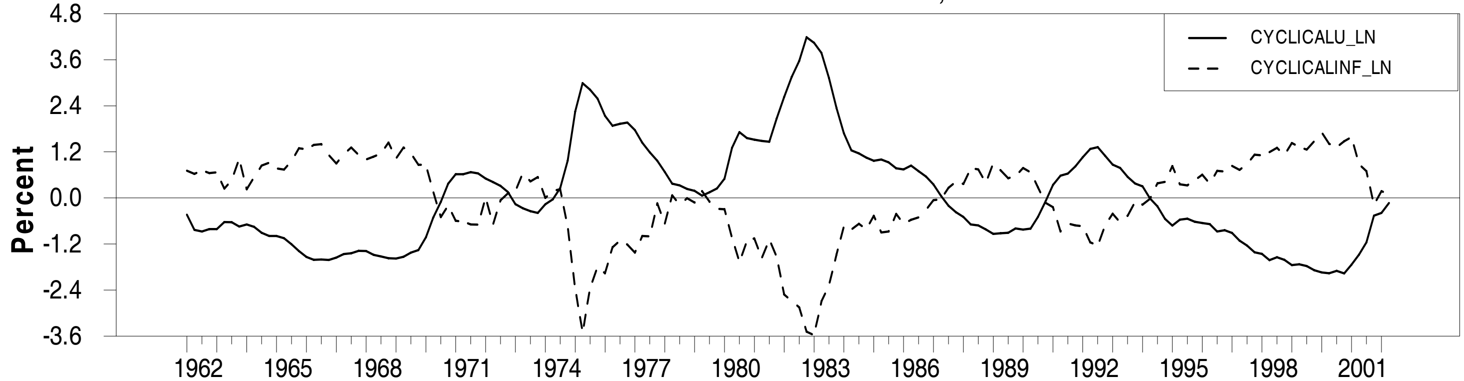


Figure 16: Cyclical Estimates in Alternative

Source: Bureau of Labor Statistics,

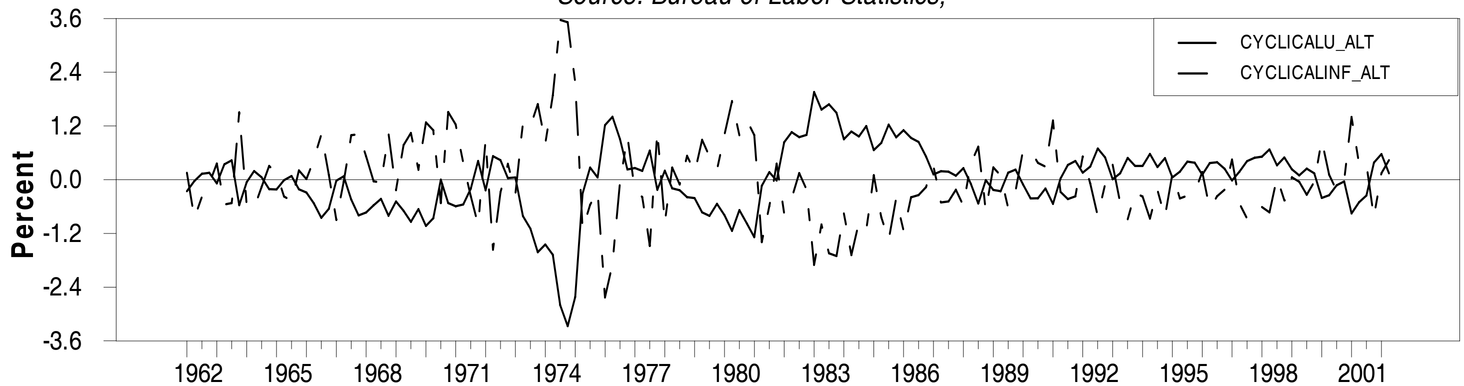


Figure 17: Cyclical Unemployment Rate Estimates

Source: Bureau of Labor Statistics,

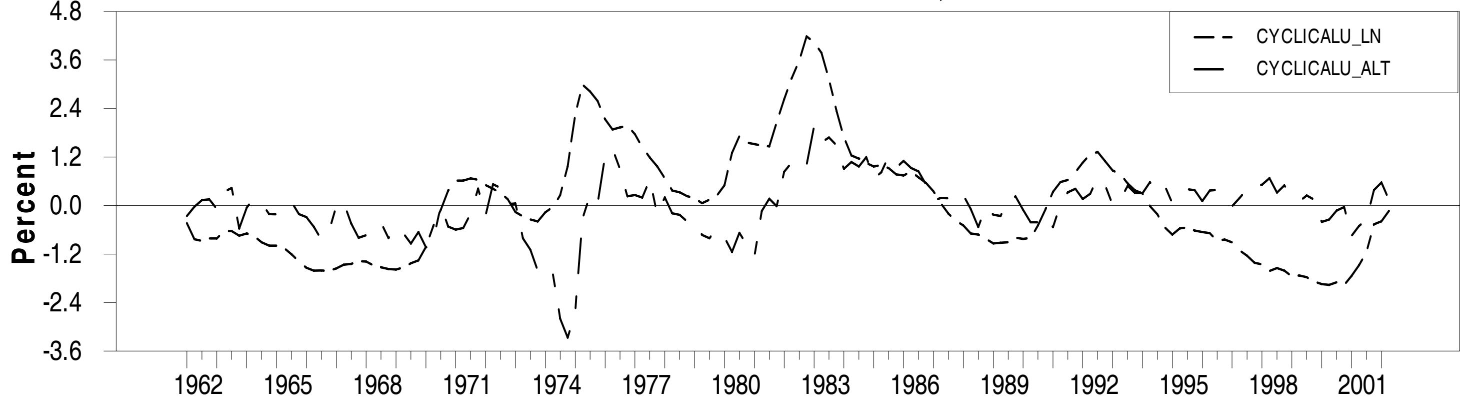


Figure 18: Cyclical Inflation Rate Estimates

Source: Bureau of Labor Statistics,