Labor Discipline, Reputation and Underemployment Traps

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Abstract

The introduction of "effort inducible" and "non-effort inducible" workers into an otherwise standard model of labor discipline produces a paradox of sorts: when firms cannot tell the difference, the predictable reductions in both output and real wages are sometimes accompanied by an increase in profits. The resolution of this paradox is found in the difference in expected productivities of workers with and without jobs, the source of a reputation effect that alters the balance of labor market power. When, as a consequence of the acquisition and depreciation of productive skills, the relative proportions of such workers are then endogenized, the model exhibits multiple equilibria for plausible parameter values. One of these equilibria can be understood as a new sort of "underemployment trap" with an atrophied primary sector.

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1 Introduction

Can firms ever benefit from incomplete information about workers’ abilities or job-related preferences? Consider an extension of the standard labor discipline model (Shapiro and Stiglitz 1984) in which firms cannot distinguish between two sorts of workers, those who could (and would, if the incentives warranted) expend some predetermined level of effective effort $\bar{e}$, and those who could not. Consistent with intuition, the expected mean productivities of those with and without jobs will then differ in a pooling equilibrium, where the difference can be interpreted as the reputation cost of job loss. Ceteris paribus, these costs increase the punishment value of dismissal and therefore alter the balance of power in labor markets. In some cases, firms will then find themselves better off without such information.

This treatment of reputation shares some similarities with Greenwald’s (1987, p. 325) model of adverse selection in labor markets, in which firms’ effort to reduce turnover rates for their better workers creates an environment in which "workers who change jobs are marked by being part of an inferior group, which lowers their future bargaining power and wages." Unlike Greenwald (1987), however, this paper features the moral hazard/effort extraction problem that firms must (also) confront. The properties of the benchmark model in the second section will perhaps also remind some of Levine’s (1988, 1989) work, in which the introduction of "just cause" policies in labor discipline models increases profits because firms are forced to internalize the negative externalities associated with overstrict dismissal policies.

To the extent that reputation costs constitute one of the "scars" (Ellwood 1982) of job loss, it becomes useful to compare the results of model calibration exercises with the empirical literature on displacement costs. Farber (2003), for example, calculates that for those who participated in one or more Displaced Worker Surveys between 1981 and 2001, the difference between pre- and post-displacement real wages for "full-time to full-time transitions" varied, from 0.9% in 1997-1999 to 12.2% in 1989-1991. (As these particular numbers hint, the dif-

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1 The earliest versions of this paper benefited from conversations with Carolyn Craven, John Geanakoplos, Ben Polak and David Weiman. It was revised while on leaves at Yale and the University of California at San Diego, with additional feedback from Jeff Carpenter. The usual disclaimers hold.

2 To be effective, the effort, however strenuous, must produce some threshold level of output.
ference is also countercyclical, so it comes as no surprise that for the most recent
(1999-2001) of Farber’s (2003) cohorts, it increased to 10.6%. Furthermore, the
data also reveal that in relative terms, less educated workers suffer more. The
importance of specific human capital to such workers implies that the sometimes
sudden depreciation of productive skills after displacement, and the need
to (re)acquire them when hired, will shape labor relations, a proposition that is
central to the extended model in the fourth section.

As Farber (2003) himself notes, however, these numbers understate the costs
of displacement: no transition is instantaneous, for example, and not all are
"full-time to full-time." On the basis of a longitudinal data set of Pennsylvanian
workers, Jacobson, Lalonde and Sullivan (1985) found that 24 months after
displacement, workers still earned 20 percent less than before, and that this
difference was persistent. These estimates more or less are consistent with
the more recent work of Schoeni and Dardia (1996), who studied workers in
California, and Stevens (1997), who examined the effects of multiple job loss.
In a similar vein, Hall (1996) concluded on the basis of Ruhm’s (1991) PSID-
based estimates that the discounted value of lost earnings is equal to 120 percent
of average annual compensation.

The next section describes a discrete time model of labor discipline in which a
small fraction of all workers do not expend effective effort, and draws attention
to one of its most important, if somewhat paradoxical, features: with the limited
information available to firms, total income falls but is redistributed so that
profits, both absolute and as a share of national income, sometimes rise. The
third section evaluates the comparative statics of the model’s unique pooling
equilibrium for a set of reasonable parameter values, and finds that in practice,
the effects on wages and employment could be substantial.

The existence of "non-effort inducible" or NE workers needs to be rationalized,
however, and this is the purpose of the fourth section, in which the proportions
of NE and effort inducible or EI workers are endogenized. In particular, it is shown that when (a) the NE/EI distinction reflects differences in
productive skills and (b) there is some likelihood that the unemployed EI lose
these skills and become NE, and some likelihood that employed NE workers will

\[3\] Effort inducible workers would of course prefer to signal this to firms, but it is not clear how this could be done. See, for example, Jullien and Picard (1998) and Albrecht and Vroman (1999).
(re)acquire them and become EI, labor markets will exhibit positive feedback and (in some cases) multiple equilibria. One of the stable equilibria can be understood as an unemployment trap or, if the model is recast as one with dual labor markets as in Bulow and Summers (1986), an underemployment trap with an atrophied primary sector. The section concludes with another comparative statics exercise, this one on the effects of variations in the rates at which human capital is acquired and lost.

Like Azariadis and Drazen (1990) or Becker, Murphy and Tamura (1990), the extended model identifies the labor market as the source of "threshold externalities." As Topel (1999) observes in his review of the literature, however, the required non-convexities can arise for many reasons, and this is perhaps the first paper in which labor discipline and reputation costs are featured.

The fifth and final section then summarizes the main results and describes some possible extensions of the model.

2 Workers, Firms and Labor Market Equilibrium: A Benchmark Model

Suppose that there are \((1 - \epsilon)H\) identical and infinite-lived effort inducible or EI workers, each with discount rate \(\theta\) and within period VNM preferences \(u(\omega, e) = \omega - e\), where \(\omega\) is the real wage and \(e\) is effort. At the start of each (discrete) period, EI workers must decide whether to expend some minimum effort level \(\bar{e}\) or to withhold their labor power, in which case \(e = 0\). There exists some likelihood \(d\) that firms will detect the absence of effort each period, and all detected workers are dismissed.4 Because the incentive condition for EI workers is satisfied in equilibrium, all EI dismissals violate the "just cause principle" (Levine 1989). A proportion \(q\) of all workers, EI and NE, who are not dismissed in a particular period are assumed to be displaced for other reasons.

To derive the incentive condition for EI workers, observe that the lifetime utilities of those who expend \(\bar{e}\) and those who do not, denoted \(V_1\) and \(V_2\), are:

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4This is less restrictive than first seems. It can be shown that within this class models, firms will choose harsh dismissal policies. For an example, see Levine (1989).
\[ V_1 = \frac{\omega - \bar{e} + \theta q V_3}{1 - \theta(1 - q)} \]  
\[ V_2 = \frac{\omega + \theta(d + q(1 - d))V_3}{1 - \theta(1 - q)(1 - d)} \]  

where \( V_3 \) is the lifetime utility of an unemployed EI worker.\(^5\) EI workers will therefore not withhold effort if \( V_1 \geq V_2 \) or, from (1) and (2), if:

\[ \omega \geq \left( \frac{1 - \theta(1 - q)(1 - d)}{\theta d(1 - q)} \right) \bar{e} + (1 - \theta)V_3 \]  

When this requirement is just met, \( V_3 \) will itself be a simple function of \( V_1 \) and the likelihood \( a \) that an unemployed worker will be (re)hired at the start of the next period:\(^6\)

\[ V_3 = \frac{aV_1}{(1 - \theta(1 - a))} \]  

Substitution for \( V_1 \) in (1) then leads to:

\[ V_3 = \frac{a(\omega - \bar{e})}{(1 - \theta)(1 - \theta(1 - a)(1 - q)(1 - f d))} \]  

This allows the incentive condition (3) to be rewritten, after considerable simplification, as:

\[ \omega = \left( \frac{1 - \theta(1 - a)(1 - q)(1 - d)}{\theta(1 - a)(1 - q)d} \right) \bar{e} \]  

This is a variant of the familiar Shapiro-Stiglitz (1984) "no shirking condition" and, as such, shares some of its characteristic properties: to induce their EI workers to expend effort, for example, firms must offer them higher wages the higher the likelihoods of rehire \( a \) or separation \( q \), or the lower the likelihood of detection \( d \).

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\(^5\)To derive (for example) the second condition, observe that the likelihood that an EI worker who withholds effort will lose her job at the end of the period is equal to \( d + (1 - d)q \), the sum of the probabilities that she is detected, \( d \), and not detected but separated for other reasons, \( q(1 - d) \). The likelihood that she remains employed is therefore \( 1 - (d + (1 - d)q) = (1 - d)(1 - q) \). Bellman’s Principle then implies that \( V_1 = \omega + \theta[(d + q(1 - d))V_3 + (1 - q)(1 - d)V_1] \) which is, after simplification, the expression in (2). The derivation of (1) follows similar lines.

\(^6\)To confirm (4), observe that an EI job seeker will be offered a job, and therefore receive \( V_1 \), with likelihood \( a \), but receive \( \theta V_3 \) with likelihood \( 1 - a \) under the assumption that wages and effort are both zero in the current period in the event of an unsuccessful search.
There is a critical difference between the two incentive conditions, however, because the presence of NE workers affects the likelihood that EI workers are rehired. If there are $N$ employed workers, both EI and NE, each period, and a fraction $\pi$ of these are NE, the number of workers who will lose their jobs each period is $[q(1 - \pi) + (d + q(1 - d))\pi]N$ and, in stationary equilibrium, the same number will be (re)hired at the start of the next.\(^7\) It follows that $(1 - d)(1 - q)\pi N$ NE and $(1 - q)(1 - \pi)N$ EI workers will retain their jobs from one period to the next, and that $H - [(1 - q)(1 - \pi) + (1 - q)(1 - d)\pi]N = H - (1 - q)(1 - \pi d)N$ workers, EI and NE, will be available for hire at the start of each period. The common likelihood of rehire $a$ is therefore:

\[
a = \frac{[q(1 - \pi) + (d + q(1 - d))\pi]N}{H - [(1 - q)(1 - \pi) + (1 - q)(1 - d)\pi]N} = \frac{(\pi d + q(1 - \pi d))n}{1 - (1 - q)(1 - \pi d)n}
\]

or the ratio of new hires to (start of period) job seekers, where $n = N/H$ is the employment rate.

This implies that for fixed $N$, the introduction of NE workers is associated with an increase in the likelihood of rehire: $(1 - q)\pi d$ more workers are hired each period,\(^8\) and the number of those without work the same amount, from $H - (1 - q)N$ to $H - [(1 - q)(1 - \pi) + (1 - q)(1 - d)\pi]N$. In the absence of any employment effects, then, EI (and for that matter, NE) workers will command higher wages. In graphical terms, the incentive condition or "supply wage relation" (Blanchard and Katz, 1997) shifts upward in $(\omega, n)$-space.

The intuition for this result is that the introduction of NE workers causes labor markets to become more turbulent: conditional on the size of its work force, each firm will hire and fire more workers each period. For the risk neutral EI worker, such turbulence is beneficial: the likelihood of rehire rises (or, from another perspective, the expected duration of joblessness falls) after a random separation.

\(^7\)The likelihood that an employed EI worker will lose her position at the end of a particular period is $q$, while the likelihood that an NE worker will is $d + (1 - d)q$. It follows that $q(1 - \pi)N$ EI workers and $(d + (1 - d)q)\pi N$ NE workers will be subtracted and (in equilibrium) added to the number of employed workers each period.

\(^8\)This is the difference between the number hired when $\pi = 0$, $qN$, and the number hired when $\pi \neq 0$, $[q(1 - \pi) + (d + q(1 - d))\pi]N$. 

6
On the other hand, the proportion of those without jobs at the end of each period who are NE, denoted $p$, will exceed the proportion, denoted $\pi$, of those with jobs who are also NE. The relative NE-abundance of the jobless pool is in turn the source of the reputation effect that undermines the post-displacement position of EI workers. To formalize this, observe that as a consequence of Bayes’ Theorem, the likelihood that a particular worker is NE, conditional on membership in the start-of-period jobless pool, is equal to the ratio of the likelihood that she is NE and jobless to the likelihood that she is jobless:9

$$
\begin{align*}
p &= \frac{\epsilon H - (1 - q)(1 - d)\pi N}{H - [(1 - q)(1 - \pi) + (1 - q)(1 - d)\pi]N} \\
&= \frac{\epsilon H - (1 - q)(1 - d)\pi N}{H - (1 - q)(1 - \pi d)N} \\
&= \frac{\epsilon - (1 - q)(1 - d)\pi n}{1 - (1 - q)(1 - \pi d)n}
\end{align*}
$$

(8)

For the number of jobless NE workers to remain constant in equilibrium, however, $p$ must meet a second condition:10

$$
\begin{align*}
p &= \pi \frac{d + q(1 - d)}{(d + q(1 - d)) + (1 - \pi)q} \\
&= \pi \frac{d + q(1 - d)}{\pi d + q(1 - \pi d)}
\end{align*}
$$

(9)

Combined, (8) and (9) define a pair of implicit functions $p = p(n)$ and $\pi = \pi(n)$ with three important properties:

1. the proportion $p = p(n)$ of those out of work at the start of each period who are NE is an increasing function of $n$, with $p(0) = \epsilon$ and $p(1) = \epsilon(d + q(1 - d))/(\epsilon d + q(1 - \epsilon d))$

9 This expression can also be motivated as a constraint on the total number of NE workers in equilibrium, and the companion to (9), which constrains the number of NE workers in the jobless pool.

10 To motivate (9), observe that there are $\pi N$ employed NE workers at the start of each period, of whom $(d + q(1 - d))\pi N$ will either be separated or detected and dismissed at the end of the same period. Since a proportion $p$ of all those in the start of period jobless pool, and therefore of all new hires, are NE, it follows that $(\pi(d + q(1 - d)) + (1 - \pi)p)\pi N$ NE workers will be hired each period. If these flows are to offset in equilibrium, then (9) must hold.
(ii) the proportion $\pi = \pi(n)$ of those employed each period who are NE is also an increasing function of $n$, with $\pi(0) = \epsilon q/(q + (1 - q)(1 - \epsilon)d)$ and $\pi(1) = \epsilon$, and

(iii) for $n \in (0, 1)$, $p(n) > \epsilon > \pi(n)$

To provide some intuition for these, observe that the third condition, the basis of the reputation effect, is consistent with the view that for each $n$, the likelihood $q$ that employed EI workers will find themselves in the jobless pool at the end of each period is smaller (perhaps much so) than the likelihood that NE workers will. If the number of EI (or NE) workers who are hired each period is to equal the number who lose their positions, then the proportion $\pi$ of employed NE workers must be less than the proportion $p$ of job seekers who are also NE. In other words, the likelihood that someone chosen at random from the jobless pool will be NE exceeds the likelihood that those now at work will be. The second condition follows from the requirement that as $n$ rises, so, too, must the proportion $\pi$ of employed workers who are NE, as more of the otherwise NE-abundant pool of job seekers is absorbed. As $\pi$ rises, however, so does the proportion $p$ of those out of work who are also NE, and this is the rationale for the first condition.

A "demand wage relation" (Blanchard and Katz, 1997) or "price-setting curve" (Layard, Nickell and Jackman, 1991) is then needed to close the model. To this end, assume that when effort is positive and effective, each new worker adds a constant $\alpha \bar{e}$ units to total output. Because new workers are recruited from the jobless pool, however, and a proportion $p(n) < \pi(n)$ of these are NE, the average product of labor, $\alpha \bar{e}(1 - \pi(n))$, declines as employment $n$ rises, and exceeds the marginal product of labor, $\alpha \bar{e}(1 - p(n))$. Firms are assumed to be imperfect competitors in product markets, and to set prices as a constant mark-up, $\mu$, over marginal labor costs.\footnote{The constant mark-up rule is a common simplification that commands limited support from empirical macroeconomists. For a review of the literature, see Woodford and Rotemberg (1999).} The demand wage relation is therefore:

$$\omega = \frac{\alpha \bar{e}(1 - p(n))}{1 + \mu}$$

(10)

A pair of representative equilibria, with and without NE workers, is depicted in Figure 1. For the reason explained earlier, the presence of NE workers shifts
the incentive condition upward, from $\mathbf{IC}_1$ to $\mathbf{IC}_2$. The demand wage curve pivots, and shifts, downward, from $\mathbf{D}_1$ to $\mathbf{D}_2$. As a result, those workers with jobs will be paid less, $\omega^*_2$ rather than $\omega^*_1$, and fewer workers, $n_2^*$ versus $n_1^*$, will have them. To understand how income is reduced but also redistributed, observe that firms' net (of labor costs) revenues per worker will be:

$$
\alpha \bar{e}(1 - \pi(n)) - \omega = \alpha \bar{e}(1 - \pi(n)) - \frac{\alpha \bar{e}(1 - p(n))}{1 + \mu} = \frac{\mu \alpha \bar{e}(1 - \pi(n))}{1 + \mu} + \frac{\alpha \bar{e}(p(n) - \pi(n))}{1 + \mu} \quad (11)
$$

If there were two sorts of workers but the sale of labor power were not "contested" (Bowles and Gintis 1993), the proportions $p$ and $\pi$ would be constant and equal, the real wage would (also) be equal to $\frac{\alpha \bar{e}(1 - \pi)}{1 + \mu}$ and the first term in (11) would be net revenues per worker. Firms do better than this, however, because of the additional benefit represented in the second term, which is the reputation cost of job loss: it is the difference in expected productivities (marked down) of those with and without jobs, $\alpha \bar{e}(1 - \pi(n)) - \alpha \bar{e}(1 - p(n)) = \alpha \bar{e}(p(n) - \pi(n))$.

Net revenues and therefore profits will rise if, as is the case here, the "per unit effect" dominates the "quantity effect." If it is workers who own these firms, household income will fall but an increased share of this reduced income will assume the form of dividends, etc.

3 Comparative Statics For A Calibrated Benchmark Model

The comparative statics of the benchmark model are not difficult to characterize in qualitative terms, but an evaluation of their practical importance requires the substitution of plausible parameter values, some of which are difficult to calibrate. To start, the rates of separation $q$ and time preference $\theta$ were set equal to 0.15 and 0.95, respectively. The former is consistent with the adjusted flow data in Summers and Poterba (1986)\textsuperscript{12} and the latter is the equivalent of a displacement rates reported in Farber (2003).

\textsuperscript{12}It is also less than the estimates in Abowd and Zellner (1985) but more than the "between DWS sample" displacement rates reported in Farber (2003).
5.2% real interest rate in the absence of financial market imperfections. In the limit case of no NE workers, the position of the horizontal demand wage schedule determines the equilibrium wage, in which case $\frac{\alpha\pi}{1+\mu} = 40$ (thousand per annum) is a reasonable choice. If labor’s share is two thirds, which implies that $\mu = 0.50$, then $\alpha\pi = 60$. We know much less about the individual parameters $\alpha$ and $\pi$ than their product, however, and still less about the likelihood of detection $d$, but the values of $\pi$ and $d$ determine the position of the incentive condition and therefore, even in the limit case, the equilibrium level of employment. Some trial and error led to $\pi = 5$ - and therefore $\alpha = 12$ - and $d = 0.50$ as sensible first choices: the equilibrium unemployment $u^*$ and rehire $a^*$ rates are then equal to 5.4% and 72.5%, respectively, where the latter corresponds to a mean (completed) jobless spell of 19.7 weeks. To compare these numbers to US data for the fourth quarter of 2004, the average unemployment rate was 5.5 percent, the average duration of unemployment was almost 20 weeks, and average earnings in goods-producing sectors were a little less than $700 per week or $36000 per year.

The comparative statics for the parameter of interest, $\epsilon$, are summarized in Table 1, which reports the equilibrium values of $\omega^*$, $u^*$ and $a^*$, as well as the equilibrium proportions $p^*$ and $\pi^*$, as the share of NE workers varies between 0 and 10 percent. It also reports the mean jobless spell, in weeks, consistent with $a^*$, the reputation cost of job loss $\frac{\alpha\pi(p(n)-\pi(n))}{1+\mu}$, defined above, the reduction in output and increase in net revenues, both expressed as a proportion of their respective values in the baseline ($\epsilon = 0$) case, as well as the jobless rates for the EI and NE sub-populations, denoted $u_{EI}^*$ and $u_{NE}^*$.14

The numbers in Table 1 suggest that labor market outcomes are quite sensitive to variations in $\epsilon$. As the share of NE workers in the labor force rises from 0 to just 2 percent, for example, the wages of employed workers fall from 40 thousand to 37.5 thousand, or by 6.25 percent. Total output falls much less than this, however, about 1.7 percent, while firms’ net revenues rise almost 7 percent, which implies that the redistribution effect dominates in practice. The

13If these numbers are indeed reasonable, the calibrated model lends some support to Juster’s (1985) view that the costs of job-related activities are often small.

14The values of $u_{EI}^*$ and $u_{NE}^*$ are equal to $\frac{1-\pi(n)\pi}{1-\epsilon}$ and $\frac{\pi(n)\pi}{\epsilon}$ respectively.
unemployment rate rises 0.7 percentage points, from 5.4 to 6.1 percent, but this obscures an important difference: the rate for EI workers is 5.8 percent, but that of NE workers is 19.1. Likewise, the likelihood of rehire falls, from 72.5 percent to 70.8, equivalent to an increase in the mean jobless spell from 19.7 weeks to 21.4.

The equilibrium values of \( p^* \) and \( \pi^* \), which are central to the characterization of labor market behavior in this framework, are 6.3 and 1.7 percent, respectively. In other words, when 2 percent of all workers cannot expend effective effort, firms will infer that more than 6 percent of all those available for hire at the start of each period have this characteristic. It follows that the expected productivities of workers with and without jobs will be \( \alpha\bar{\sigma}(1-\pi(n)) = 59.0 \) and \( \alpha\bar{\sigma}(1-p(n)) = 56.2 \) thousand, so that the reputation cost of job loss, \( \frac{\alpha\bar{\sigma}(\bar{p}(\alpha)-\pi(n))}{1+p} \), is more than $1800, or four and a half percent of the representative EI worker’s wages. This is smaller than the estimates listed in the introduction, but the assumed proportion of NE workers is also small, and unlike the extended model in the fourth section, this calculation does not include the possible depreciation of human capital after displacement.

As the proportion of NE workers is increased to 10 percent, the redistribution effect becomes even more pronounced: output falls (just) 8.3 percent relative to the "no NE" baseline, but wages fall more than 25 percent, from 40 thousand to 29.6 thousand. The net revenues of firms, on the other hand, increase 21.7 percent relative to the same baseline. The overall, EI and NE unemployment rates are now 9.4, 7.7 and 24.3 percent, respectively, while the common likelihood of rehire falls to 64.2 percent, consistent with an increase in the mean jobless spell to 28.9 weeks or between 6 and 7 months. The equilibrium values of \( \pi^* \) and \( p^* \) increase to 8.4 and 25.9 percent, so that the reputation cost of job loss is about $7000 or almost 25 percent of the now reduced wage rate, a number much closer to current estimates.

As alluded to earlier, however, there are few empirical studies to motivate the choice of the detection rate \( d \) and the cost of effort \( \bar{e} \) (or, if one prefers, output per unit of effort \( \alpha \)) so that some sort of robustness check is needed. To this end, Figures 2(a)-2(c) plot the equilibrium wage \( \omega^* \), jobless rate \( \pi^* \) and reputation effect \( R^* \) for values of \( d \) between 0.25 and 0.75 and \( \bar{e} \) between 2.5
and 7.5, with the product $\alpha \bar{e}$ fixed at 60 for $\epsilon = 0.02$ or 2 percent.\footnote{The intervals are equal to the initial values plus or minus 50 percent.}

[Insert Figures 2(a), 2(b) and 2(c) About Here]

Figure 2(a) reveals that once $\alpha \bar{e}$ is fixed, the "price effects" of variations in either the likelihood of detection or cost of effort will be small in practice. As expected, for example, the real wage $\omega$ is an increasing function of $\bar{e}$, but for the initial choice of $d (= 0.50)$, it rises from 37.3 thousand when $\bar{e} = 2.5$ to just 37.7 thousand when $\bar{e} = 7.5$. The effects of variations in $d$ are not much more dramatic: for $\bar{e} = 5$, the wage falls from 38.4 thousand when the detection rate is 25 percent to 36.6 thousand when it is 75 percent.

As Figure 2(b) shows, however, the "quantity effects" are substantial. At the benchmark detection rate, for example, an increase in the cost of effort from 2.5 thousand to 7.5 causes the jobless rate to rise from 2.8 percent to 9.9. In a similar vein, as the likelihood of detection rises from 25 to 75 percent, it falls from 11.1 to 4.3 percent.

\section{4 Positive Feedback and Multiple Equilibria in an Extended Model}

There are at least two sorts of explanation for the existence of NE workers. The first follows from the observation that unemployed EI workers will sometimes become NE. The simplest reason for this is that separation causes firm- or sector-specific human capital to be lost, a phenomenon explored in the empirical work of Hamermesh (1987) and Topel (1990). For displaced workers who are not rehired soon, the slow(er) erosion of more portable skills would produce similar results. The adverse psychological effects of joblessness could also explain such transformations: Darity and Goldsmith (1996), for example, find that unemployment produces measurable damage to workers' well-being and that mental health is an important determinant of productivity. In a similar vein, Oswald (1997) concludes that joblessness is a source of substantial "non-pecuniary distress." If, in the present context, all of these are understood in terms of an increased likelihood of "failure," where failure means that effort
is ineffective, some workers will find it rational, in the sense of (3), to withhold effort altogether.

A second, and quite different, explanation turns on the identification of NE workers as those with substantial extra-market opportunities and/or wealth: the better a worker’s default position, the smaller the punishment value of dismissal and therefore the better the incentives that firms must provide to induce the required effort level.

To sketch an extension of the model based on the first of these, suppose that EI workers are as before but that NE workers now have preferences \( u(\omega, e) = \omega - ke \), where \( k > 1 \) is chosen so that, for all "reasonable" values of \( \omega \), \( e = 0 \) is optimal: NE workers, in other words, find it more difficult, if not impossible, to provide the required level of effective effort \( \bar{e} \). Suppose, too, that there is some likelihood \( z_2 \) that an unemployed EI worker will become NE after each period spent in the jobless pool, but some likelihood \( z_1 \) that an employed NE worker will become EI, despite the absence of effective effort.\(^{16}\) (Strictly speaking, the second is less "learning by doing" than "learning by observing." ) In heuristic terms, \( z_1 \) and \( z_2 \) are the rates at which workers reskill and deskill.

Even with attention restricted to stationary pooling equilibria, the characterization of labor market flows - in particular, the determination of \( z_1, z_2 \) and the now endogenous \( \epsilon \) - becomes more complicated. From the schematic in Figure 3, for example, the number of EI workers with jobs will increase for two reasons: \( a(1 - p)S \) EI workers will be hired from the jobless pool, where \( S \) is the number of those out of work at the start of each period, before firms have replaced their "lost" workers, and a fraction \( z_1 \) of the \( (1 - q)(1 - d)\pi N \) NE workers with jobs who were neither separated nor dismissed in the previous period will have reskilled and become EI. It will decrease, on the other hand, because a fraction of \( q \) of the \( (1 - \pi)N \) EI workers employed each period will be displaced. For the number of EI workers with jobs to remain constant in equilibrium, then, it must be that:

\(^{16}\)It is also assumed that the number of workers is sufficient to treat expected and actual flows as equal.
\[ a(1-p)S + z_1(1-q)(1-d)\pi N = q(1-\pi)N \]  \hspace{1cm} (12)

where:

\[ S = H - [(1-\pi)(1-q) + (1-d)(1-q)\pi]N = H - (1-q)(1-\pi d)N \]  \hspace{1cm} (13)

is one measure of the number of job seekers.

Likewise, for the number of NE workers with jobs, and the number of EI workers without, to remain constant from one period to the next, it must be that:\textsuperscript{17}

\[ apS = [z_1(1-q)(1-d) + (q + d(1-q))\pi]N \]  \hspace{1cm} (14)

and:\textsuperscript{18}

\[ a(1-p)S = q(1-\pi)N - z_2(1-a)(1-p)S \]  \hspace{1cm} (15)

It is then convenient to replace either (12) or (14) with a linear combination of the two:

\[ a = \frac{[(q + d(1-q))\pi + q(1-\pi)]N}{S} \]  \hspace{1cm} (16)

which then defines the likelihood of rehire in the extended model.

Substitution for the likelihood of rehire in (12) then implies that:

\[ p = \frac{z_1(1-q)(1-d)\pi + (q + d(1-q))\pi}{(q + d(1-q)) + q(1-\pi)} \]  \hspace{1cm} (17)

which establishes an important relationship between \( p, \pi \) and \( z_1 \) that, perhaps surprisingly, does not depend on the total number of workers \( N \) employed each period. Substitution for both \( a \) and \( S \) in (15), on the other hand, leads to:

\textsuperscript{17}If (12), (14) and (15) are satisfied, then, as a matter of addition, the number of unemployed NE workers will also remain constant.

\textsuperscript{18}The first of these requires that the number \( apS \) of NE workers hired each period must offset the number of NE workers who are either displaced or dismissed and the number of NE workers who are neither but become EI at the end of the period. The second asserts that the number of EI workers who are hired, and therefore leave the jobless pool, each period must equal the sum of the number of EI workers who are displaced and the number of EI workers who are deskillled.
\[ z_2 = \frac{pq(1 - \pi)N - (1 - p)(q + d(1 - q))\pi N}{(1 - p)(H - N)} \] (18)

Given the likelihoods \( z_1 \) and \( z_2 \) that workers are reskilled and deskilled and the number of workers with jobs \( N \), (17) and (18) determine the proportions \( \pi \) and \( p \) of NE workers with and without jobs consistent with flow equilibrium, and (16) then determines the likelihood of rehire \( a \). There is no stock condition for the proportion \( \epsilon \) of NE workers in the labor force as a whole because its value is now determined within the model:

\[ \epsilon = \frac{pS - (1 - q)(1 - d)(1 - z_1)\pi N}{H} \] (19)

(As a matter of definition, a proportion \( p \) of the number of job seekers \( S \) at the start of each period are NE, while the number of those who retain their jobs from one period to the next is \([1 - z_1(1 - q)(1 - d) - (q + d(1 - q))]/\pi N\) or, after simplification, \((1 - q)(1 - d)(1 - z_1)\pi N\). The sum of these is therefore equal to the total number of NE workers, \( \epsilon H \).)

It is still the case that \( p > \epsilon > \pi \) for each \( N \), but the values of \( p \) and \( \pi \) consistent with (17) and (18) are now decreasing in \( N \). In other words, as the number of those with jobs \( N \) rises, the proportion \( \pi \) of these who are NE, as well as the proportion \( p \) of job seekers who are, will rise, too. This implies that the demand wage relation will slope upward. Labor markets now exhibit positive feedback: as the volume of employment \( N \) rises, the share of the labor force that is NE falls as some who would have otherwise remained (re)acquire productive skills. This in turn pulls down the proportions \( \pi \) and \( p \) of those with and without jobs who are NE which, under these conditions, is sufficient to "deconvexify" production.20

[Insert Figures 4a and 4b About Here]

Viewed from a somewhat different perspective, each new hire now produces positive externalities. A proportion \( p \) of these hires will be NE and, of these, a

\[^{19}\text{To see this, observe that (17) implies that } p \text{ and } \pi \text{ must rise and fall together, and that (18) implies that as both rise, } N \text{ must fall, and vice versa.}\]

\[^{20}\text{Under normal conditions, the existence of a countercyclical mark-up and/or procyclical "real marginal cost" causes the wage demand relation to slope downward. If either or both were present here, intuition suggests that the relation would be hump-shaped: for small values of } N, \text{ the "reskill effect" would dominate, but that as } N \text{ approaches } H, \text{ the mark-up and/or returns effects would. If so, the essential properties of the model would be unaffected.}\]
proportion \(1 - z_1\) will remain so from this period to the next, and even if some, perhaps most, of these are dismissed or displaced in the current period, some will continue in their jobs. The remainder, or a proportion \(z_1\) of new NE hires, will be reskilled, but of these, \(q\) percent will nevertheless return to the jobless pool at the end of the period, where other firms will hire some of them, and therefore reap the benefits, but not bear the costs, of their investment in skills. (Because firms also retain a fraction \(1 - q\) of these reskilled workers, most of the benefits are captured, however.)

To derive the incentive condition for EI workers, observe first that the lifetime utilities \(V_{1EI}^E\) and \(V_{2EI}^E\) of workers who are effort inducible at the start of a particular period are the same, mutatis mutandis, as in the benchmark model, so that (3) still constrains firms’ offers to such workers. The calculation of \(V_{3EI}^E\), however, the welfare of a (for the moment, at least) EI worker in the jobless pool is more involved. The EI worker who is displaced at the end of a particular period will now receive a job offer, and therefore \(V_{1EI}^E\), with likelihood \(a\); will not receive an offer but remain EI, and so expect \(\theta V_{3EI}^E\), with likelihood \((1 - a)(1 - z_2)\); and, most important, will neither receive an offer nor remain EI - that is, be deskilled - with likelihood \((1 - a)z_2\), the value of which is \(\theta V_{3NE}^E\), where \(V_{3NE}^E\) is the lifetime utility of an NE worker who is (currently) without a job. It follows, therefore, that:

\[
V_{3EI}^E = \frac{aV_{1EI}^E + (1 - a)z_2\theta V_{3NE}^E}{1 - \theta(1 - a)(1 - z_2)} \tag{20}
\]

To calculate \(V_{3NE}^E\), observe that NE workers without jobs will find a position, and receive \(\theta V_{2NE}^E\) - not, it is important to note, \(\theta V_{1NE}^E\), since it was assumed that NE workers choose not to expend effort, which implies that \(V_{2NE}^E > V_{1NE}^E\) - with likelihood \(a\), but not find a job, and thus receive \(\theta V_{3NE}^E\), with likelihood \(1 - a\), from which it can be inferred that:

\[
V_{3NE}^E = \frac{aV_{2NE}^E}{1 - \theta(1 - a)} \tag{21}
\]

The value of \(V_{2NE}^E\) is in turn a function of \(V_{1EI}^E\), \(V_{2EI}^E\) and \(V_{3NE}^E\):

\[
V_{2NE}^E = \frac{\omega + \theta z_1[(d + q(1 - d))V_{3EI}^E + (1 - d)(1 - q)V_{1EI}^E] + \theta(1 - z_1)(d + q(1 - d))V_{3NE}^E}{1 - \theta(1 - q)(1 - d)(1 - z_1)} \tag{22}
\]
the derivation of which involves no new complications. The NE worker with a job receives $\omega$ (not $\omega - \bar{e}$) in the current period, but with some likelihood $z_1(d + q(1 - d))$, for example, she will reskill but be detected (and then dismissed) or displaced for other reasons and so receive $V_{3EI}^{EI}$ at the start of the next period, and so on.

Combined, (1), (20), (21) and (22) comprise four linear equations in four unknowns - $V_{1EI}^{EI}$, $V_{3EI}^{EI}$, $V_{2NE}^{NE}$ and $V_{3NE}^{NE}$ - and the substitution of the solution for $V_{3EI}^{EI}$ into the incentive condition for EI workers provides the required modification of (5).

Figure 4a, introduced earlier, depicts the representative incentive and demand wage schedules in the three equilibrium case. For $z_1 = 0.90$ and $z_2 = 0.10$, for example, there is a stable interior equilibrium at which $n = 93.7$ percent (or $u = 6.3$ percent), an unstable equilibrium at which $n = 22$ percent (or $u = 78$ percent) and a stable corner equilibrium ($n = 0$ percent or $u = 100$ percent).

The last of these equilibria is not implausible, however, if the model is recast as one with dual labor markets à la Bulow and Summers (1986). In their model, most of those who do not find work in the primary market, in which effort is difficult to monitor, are absorbed into the secondary market, in which it is not. Viewed from this perspective, the corner equilibrium should be interpreted as one in which everyone works in the secondary market, rather than one in which, implausibly, no one is employed. If so, the relative sizes of these two markets is less determinate than often supposed. In particular, a predetermined set of preferences, endowments and methods of production can be consistent, both in principle and in practice, with either a vibrant or dormant high wage sector.

This pattern is reminiscent of the earliest neoclassical (Solow 1956) models of underdevelopment traps, in which the feedback mechanism assumes the form of an intensive production function that is concave for capital-labor ratios below some threshold and convex above it. In the tradition of Azariadis and Drazen (1990) and others, the model described here identifies the labor market as one source of such non-convexities. In particular, if the volume of primary sector employment is smaller than the threshold associated with the unstable equilibrium - in the case where $z_1 = 0.90$ and $z_2 = 0.10$, 22 percent of the labor force - the number of workers who could and would exert effort level $\bar{e}$ will be too small for the expected contribution of the last (or next) hire to exceed the
incentive wage for such workers. If, as a result, employment falls, however, still more workers are deskilled, and the expected marginal product of new hires falls even further and, more important, faster than the incentive that EI workers require. In this environment, a state-sponsored "big push" (Murphy, Shleifer and Vishny, 1989) might be needed to ensure that employment in this sector reaches critical mass or, to invoke Rostow’s (1960) famous metaphor, "takes off."

It is important to remember, however, that the existence of three equilibria - and, in particular, the existence of a stable equilibrium in which some, perhaps most, of the labor force is employed in the primary sector - is not assured. If, for example, the reskill rate remains equal to 90 percent but the likelihood that workers are deskilled rises to, say, 50 percent, the relative positions of the incentive and labor demand schedules are those in Figure 4(b), in which case there is just one stable corner equilibrium, at which \( u = 100 \) percent. Under these conditions, no primary labor market is (ever) viable. The reason for this is the increased, and now substantial, likelihood that EI workers who find themselves in the jobless pool are deskilled and therefore transformed into NE workers: when the number of primary sector workers is small, there are more NE workers in the jobless pool, and the expected contribution of new hires is low, but as primary sector employment increases, the decrease in the proportion of NE workers is more than offset by the increase in the effort compatible wage.

Figure 4(b) is reminiscent of Mankiw’s (1986) representation of financial market collapse. This is not a coincidence, of course: in the presence of adverse selection in credit markets, an increase in interest rates is sometimes associated, for the reasons first described in Stiglitz and Weiss (1981), with an increase in the riskiness of loans and under some conditions, there will be no interest rate at which the expected return is sufficient for banks to lend. The breakdown of the labor, rather than credit, market depicted in Figure 4(b) embodies a similar logic: there is sometimes no employment level at which the expected return on a new hire exceeds the wage required to induce effective effort.

To return to the "normal" case, it is useful to characterize the (local) comparative statics of the stable interior equilibrium with respect to the reskill and deskill rates. Consider, for example, the effects of an increase in the likelihood that EI workers in secondary sector jobs are deskilled \( z_2 \). On one hand, the
modified demand wage relation shifts downward since, for each $N$, the proportion $p$ of all workers without such jobs who are NE will increase. Consistent with Figure 3(a), this exerts downward pressure on both wages and employment. On the other hand, the modified incentive condition also shifts downward, since the EI worker who withholds effort now risks detection, dismissal and an increased likelihood that she will be deskilled, which increases the cost of job loss for fixed $N$. This in turn would tend to increase employment and, because of the positive feedback mechanism built into the primary sector, the equilibrium wage rate. The net effects on $\omega$ and $N$ depend, in other words, on the relative sizes of these shifts, which are difficult to predict a priori.

To determine which shift dominates in practice, Figures 5(a), 5(b) and 5(c) plot the equilibrium wage, unemployment rate and reputation cost of job loss for reskill rates $z_1$ between 0.5 and 1 and deskill rates $z_2$ between 0 and 0.25. To anchor the discussion that follows, the pair ($z_1 = 0.9, z_2 = 0.1$) will serve as a benchmark of sorts: in this case, $\omega = 36.1$ thousand, $u = 6.3$ percent and $R = 3.3$ thousand.

Two features stand out. First, it is clear that the first (demand wage) shift dominates: as the deskill rate rises, ceteris paribus, the wage rate falls and unemployment rises. In more evocative terms, the more rapid depreciation of human capital has a more substantial effect on productivity, loosely defined, than bargaining power. Second, the equilibrium values appear to be sensitive to variations in the deskill rate. As $z_2$ increases from 10 to 25 percent, for example, the wage falls 20 percent, to 29.7 thousand, while the unemployment rate rises from 6.3 to 8.6 percent. Likewise, the reputation cost of job loss increases from a little more than 9 percent of annual wages to almost 30 percent, or 8.5 thousand.

The net effect of variations in the reskill rate is also the result of competing shifts of the demand wage and incentive conditions. An increase in $z_1$ will reduce the proportion $p$ of job seekers who are NE at each $N$ and so causes the demand wage relation to shift upward, which tends, ceteris paribus, to increase both $\omega$ and $N$. It will also cause the incentive condition to shift upward, however, because there is an increased likelihood that workers who are displaced
and deskill will (re)acquire these skills, which then reduces the punishment value of dismissal. Unlike the case of the deskill rate, however, the two shifts more or less offset one another. As the reskill rate is reduced from 90 to 50 percent, for example, the equilibrium wage falls from 36.1 thousand to just 34.1, the unemployment rate rises from 6.3 percent to 7.0, and percent, and the reputation cost of job loss rises from 3.3 thousand (or 9 percent of compensation) to 4.6 thousand (or 13 percent).

To provide still another perspective on these results, Figure 5(d) plots the now endogenous proportion of the labor force that is NE. For the benchmark values $z_1 = 0.90$ and $z_2 = 0.10$, for example, 9.8 percent of all workers are NE. As the reskill rate decreases to 50 percent (but the deskill rate remains the same) the proportion rises to 15.2 percent. When the reskill rate is held fixed, however, and the likelihood that workers are deskill rises to 25 percent, 26 percent of the labor force will be NE.

Accepted at face value, these data contain at least two lessons for macroeconomists. First, that the destruction and formation of human capital are important medium run processes, that is, even when the net rate of human capital accumulation is zero. Second, that within this context, the loss of productive skills should receive at least as much attention as their acquisition: on the basis of these results, labor market outcomes are more sensitive to the former than the latter.

Last, it is also possible to calculate, for the same values of $z_1$ and $z_2$, the volume (expressed as a fraction of the labor force) of primary sector employment associated with the unstable equilibrium. (Given the correspondence principle, the data are better interpreted in terms of the minimum viable mass of the primary sector in different economies, and not as variations in this mass.)

Consistent with intuition, Figure 6 reveals that economies in which the likelihood that displaced workers are deskill is lower have a smaller viable mass. (There is also more primary sector employment in the stable equilibrium, consistent with Figure 5(b).) If the deskill rate increases much more than this, of course, both interior equilibria are lost. Figure 6 also confirms that similar benefits - better stable equilibrium, smaller minimum viable mass - also accrue to economies in which more recent hires are reskilled.
For the benchmark values $z_1 = 0.90$ and $z_2 = 0.10$, this threshold is equal to 21.7 percent of the labor force, and for a smaller reskill rate ($z_1 = 0.50$), it increases to 28.2 percent. When the reskill rate is held fixed at 90 percent, however, and the deskill rate is increased to 25, then the minimum viable mass is almost half (47.9 percent) of the labor force.

5 Conclusion

The proposition that the presence of non-effort inducible workers will somehow benefit firms seems counterintuitive until is recalled that in practice, the punishment value of dismissal often comprises a reputation cost. If a small number of such workers are introduced into the standard labor discipline model, and these workers cannot be distinguished from their effort inducible peers, a reputation effect, in the form of a difference in the mean productivities of workers with and without jobs, is established, a variation on Greenwald (1987). As a result, the number of employed workers and total output both fall, but non-labor income rises, both absolutely and as a share of national income. If the depreciation and acquisition of productive skills are made endogenous, labor markets will exhibit positive feedback and multiple equilibria for plausible parameter values. Under these conditions, the stable high employment equilibrium appears to be sensitive to variations in the rate at which jobless workers lose their skills, a result that underscores the importance of recent empirical work on displacement.

A natural extension of the model would reinterpret its labor market(s) in regional, or perhaps urban, terms. In economies where the costs of (sub-national) migration are not trivial, this framework allows, at least in principle, for the co-existence of vibrant and stagnant primary labor markets. In some cities, the model hints, the secondary, or even shadow, labor market will dominate precisely because it dominated in the past: the previous depreciation of human capital is an obstacle to firm relocation, despite the low(er) costs of labor. If a sufficient number of firms could be persuaded to relocate, however, the accumulation of productive skills would make relocation profitable. To escape these localized underdevelopment traps, a "big push" may be needed to establish a viable primary sector. What form this push should take - whether investment in "job market skills" or tax incentives to lure, and then retain, new firms - is not clear, however.
The implications for some less developed economies are similar. In cases where no vibrant primary sector exists, the explanation may be that no such sector has ever existed - so that the productive skills that would rationalize substantial domestic or foreign investment have not been accumulated - not that it could not exist.

6 References


Table 1. The Comparative Statics of Non-Effort Inducible (NE) Workers in the Benchmark Model

<table>
<thead>
<tr>
<th>ε = 0%</th>
<th>ε = 2%</th>
<th>ε = 4%</th>
<th>ε = 6%</th>
<th>ε = 8%</th>
<th>ε = 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>ω</td>
<td>40.0</td>
<td>37.5</td>
<td>35.2</td>
<td>33.2</td>
<td>31.3</td>
</tr>
<tr>
<td>u</td>
<td>5.4</td>
<td>6.1</td>
<td>6.8</td>
<td>7.6</td>
<td>8.5</td>
</tr>
<tr>
<td>a</td>
<td>72.5</td>
<td>70.8</td>
<td>69.2</td>
<td>67.6</td>
<td>65.9</td>
</tr>
<tr>
<td>Weeks</td>
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<td>21.4</td>
<td>23.1</td>
<td>24.9</td>
<td>26.9</td>
</tr>
<tr>
<td>p</td>
<td>6.3</td>
<td>11.9</td>
<td>17.1</td>
<td>21.7</td>
<td>25.9</td>
</tr>
<tr>
<td>π</td>
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<td>3.4</td>
<td>5.1</td>
<td>6.7</td>
<td>8.4</td>
</tr>
<tr>
<td>Rep</td>
<td>1.8</td>
<td>3.4</td>
<td>4.8</td>
<td>6.0</td>
<td>7.0</td>
</tr>
<tr>
<td>Lost Q</td>
<td>1.7</td>
<td>3.5</td>
<td>5.2</td>
<td>6.7</td>
<td>8.3</td>
</tr>
<tr>
<td>Profits</td>
<td>6.9</td>
<td>12.2</td>
<td>16.4</td>
<td>19.7</td>
<td>21.7</td>
</tr>
<tr>
<td>u_{EI}</td>
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<td>4.9</td>
<td>5.2</td>
<td>5.5</td>
<td>5.8</td>
</tr>
<tr>
<td>u_{NE}</td>
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<td>14.3</td>
<td>15.1</td>
<td>16.0</td>
<td>16.8</td>
</tr>
</tbody>
</table>

Notes: The definitions of ω, u, a, p, π, u_{EI} and u_{NE} are provided in the text and, with the exception of the annual real wage ω, which is measured in thousands of dollars, are reported in percentage terms. Weeks is the implied mean length of jobless spells, Rep is the reputation cost of displacement, expressed in thousands of dollars, Lost Q is the percentage decrease in total output relative to the baseline (ε = 0%) case, and Profits is the percentage increase in total non-labor income relative to the same baseline.
Figure 1. Labor Market Equilibria with and without Non-Effort Inducible (NE) Workers
Figure 2a. Real Wages (In Thousands Per Year) in the Benchmark Model As a Function of Detection Rate and Required Effort Level
Figure 2b. Unemployment Rate in the Benchmark Model
As a Function of Detection Rate and Required Effort Level
Figure 2c. Reputation Cost of Job Loss (In Thousands) in the Benchmark Model As a Function of Detection Rate and Required Effort Level
Figure 3. Labor Market Flows In The Extended Model
Figure 4a. Multiple Equilibria in the Extended Model (Standard Case)
Figure 4b. Collapse of the Primary Labor Market In Extended Model
Figure 5a. Real Wages (In Thousands Per Year) in Extended Model
As a Function of Deskil and Reskill Rates
Figure 5b. Unemployment Rate in Extended Model
As a Function of Deskill and Reskill Rates
Figure 5c. Reputation Cost of Job Loss (Thousands) in Extended Model
As a Function of Deskill and Reskill Rates
Figure 5d. Share of Non-Effort Inducible (NE) Workers (Percentage of the Labor Force) in Extended Model As a Function of Deskil and Reskill Rates
Figure 6. Minimum Viable Mass (Primary Sector Employment As A Percentage of the Labor Force) in Extended Model As a Function of Deskill and Reskill Rates