INCREASING THE "POP" IN POPCORN

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ABSTRACT

Our goal in this project is to increase the size of popcorn flakes, which can make the snack more desirable for consumers, as well as more profitable for producers. We proceeded by modeling the popping process as an adiabatic expansion and lowering the pressure in the popping chamber to increase the flake size. To test this theory, we experimentally determined the effect of decreased pressure on popcorn size in three different apparatuses: a stove popper, a movie popper, and a microwave. Our results indicate that there will be an increase in flake volume and a decrease in waste when popcorn is popped under reduced pressure.

INTRODUCTION

Popcorn is one of the most popular snacks in America. To meet the demand for this snack food, the species of corn, *Zea mays averta,* is now specifically cultivated. The hard, hydrophobic outer shell and starch-filled kernel make it ideal for popping. Consumed at movie theaters, at home, at sporting events, and produced in such variations as caramel corn and kettle corn, popcorn is sold today at an approximate rate of 17 billion quarts per year (1).

Its earliest roots were in Central and South America where Native Americans popped wild and cultivated corn in heated sand for consumption. In the 16th century, the Spanish observed that popcorn was not only regularly eaten by the Aztecs, but also used for necklaces, headdresses, and other decorative and religious purposes. The corn, dubbed *momochitl* after being dried, was used to pay tribute to Aztec gods, including Tialoc, the god of rain and fertility. Popcorn has gradually evolved from the early technique of popping corn in heated sand to modern methods of popping popcorn that exist around the globe (1). A great advance occurred late in the 19th century when Charles Cretors' mobile, steam-powered popcorn machine replaced the method of popping on a wire mesh. Since the new method led to more evenly heated kernels, it became the most popular way of cooking popcorn. By mixing lard, butter, and salt with the corn before heating it, Cretors' technique set the stage for popcorn to dominate the snack food industry. During the Great Depression, popcorn's popularity boomed in America when bags of popcorn were sold competitively for cheap prices, ranging from five to ten cents. Since then, popcorn has remained as an iconic part of American life, adapting itself to technological advances such as the television and microwave. It also became an important part of the food industry, as companies continued to try to meet the demand for this popular snack food (2).

Maximizing profits and increasing the public appeal of popcorn are the main goals of popcorn manufacturers. To compete with popcorn and snack manufacturers, companies must attempt new methods of cultivating and popping corn. Three standards that industry established now play a major part in governing the quality of popcorn: the expansion value (σ), the flake size (π), and the amount of waste (ω). These values deal with measurements of volume and quantities of flakes and kernels. The optimization of such values may be achieved in a variety of ways. The designing of an apparatus that can perform such a task would be a definite interest of popcorn companies that wish to amplify the quality of their popcorn and thus increase their competitiveness in the modern economy.

THEORY

In order to correctly analyze the best way to create larger popcorn flakes, the actual process of popping popcorn kernels must be examined. Corn kernels consist of a starch and moisture mixture surrounded by a hard shell called the pericarp. During the popping process, externally added heat allows the moisture inside to mix uniformly with starch until the temperature inside the kernel exceeds the boiling point of water. At this instant, the trapped moisture vaporizes and expands. The pressure inside the shell increases until it exceeds the yield pressure, the pressure that the pericarp can withstand before it breaks. When the pericarp breaks, the water vapor and starch expand into the surroundings. Once the water vapor pressure reaches the pressure of the popping chamber, the starch mixture stops expanding and solidifies. The starch from the popcorn stops expanding once the water vapor pressure drops to the surrounding pressure. Since the expansion after the pop is extremely rapid, no heat is transferred between the kernel and its surroundings. Therefore, the inflation can be thermodynamically modeled as an adiabatic expansion (3).

In an adiabatic expansion, the volumes and pressures in the process of popping popcorn relate as follows:

$$P_i V_i^{\gamma} = P_f V_f^{\gamma} = C_0 \ , \tag{1}$$

where P_i is the yield pressure or the pressure on the pericarp of the kernel immediately before it pops, V_i, the initial volume of the kernel, V_f, the final volume of the flake, and P_f, the pressure surrounding the flakes after the kernels pop. Under normal conditions, P_f is 1 atm. γ is the ratio of the specific heat at constant pressure to the specific heat at constant volume. In this experiment, the value of γ is 1.33, which is the accepted value for water vapor. (3)

In order to maximize final volume, Eq. (1) is solved for V_f to obtain:

$$V_f = V_0 \left(\frac{P_i}{P_f} \right)^{\frac{1}{\gamma}} \ . \tag{2}$$

This equation shows that lowering P_f increases V_f. Theoretically, the flake size can be increased by decreasing the pressure of the kernel's surroundings.

In order to evaluate the effect of lowering the pressure, the following three variables used by industry are defined:

$$\sigma = \frac{\text{total popped volume (cm}^3\text{)}}{\text{original sample weight (g)}} \tag{3}$$

$$\pi = \frac{\text{total popped volume (cm}^3\text{)}}{\text{number of popped kernels}} \tag{4}$$

$$\omega = \frac{\text{number of unpopped kernels}}{\text{original number of kernels}} \times 100\% \tag{5}$$

These symbols indicate the expansion volume, the size of each flake, and the percentage of waste respectively. Under ideal conditions, industries have obtained values as high as σ = 45 cm^3/g and π = 8 cm^3/kernel, while waste has been as low as ω = 6.8%. On the other hand, consumers usually obtain values of σ = 36 – 40 cm^3/g, π = 5 – 7 cm^3/kernel, and ω = 10 – 12 %. By lowering the surrounding pressure, we predicted that σ and π will increase while ω will decrease (4).

In this experiment it was assumed that the popcorn kernels and flakes are spherical in shape, and that the water vapor is an ideal gas. This means we are assuming the vapor particles within the kernel are infinitely small and do not interact with each other.

Starting with the Eq. (1), the instantaneous pressure is isolated to obtain

$$P = P_0 \left(\frac{V_0}{V} \right)^{\gamma}, \tag{6}$$

where P_0 = initial yield pressure, V_0 = initial kernel volume, and V is the final volume. Since the kernel is modeled as a sphere, its volume is $V = (4\pi/3)R^3$, where R is the radius of the kernel. Substituting this volume expression for V into Eq. (6) we find:

$$P = \frac{P_0 \left(\frac{3V_0}{4\pi} \right)^{\gamma}}{R^{3\gamma}}. \tag{7}$$

Reference (3) suggested that the rate of change of the radius with respect to time will be proportionate to the pressure gradient such that

$$\frac{dR}{dt} = k\left(P - P_f\right), \tag{8}$$

where k is proportionality constant, P is the instantaneous pressure, and P_f is the surrounding pressure. The expression for P as a function of radial length is Eq. (7) was substituted into Eq. (8). The variable k depends on several factors, including the amount of moisture in a kernel. The proportionality constant does not affect the final size of the popcorn flake, but it does affect the growth speed of the kernel. The larger the k, the faster the popcorn flake will reach its asymptotic size. Also, P_f is considered insignificant compared to the instantaneous pressure exerted by the kernel as it pops. Thus, the following approximation for the change in radius with respect to time was used:

$$\frac{dR}{dt} = k\frac{P_0\left(\frac{3V_0}{4\pi}\right)^{\gamma}}{R^{3\gamma}}. \tag{9}$$

With some calculus and algebraic manipulation, we used the following steps to obtain an expression for the radius at any given time during the popping process:

$$R^{3\gamma}\,dR \approx kP_0\left(\frac{3V_0}{4\pi}\right)^{\gamma} dt \tag{9a}$$

$$\int_{R_0}^{R} R^{3\gamma}\,dR \approx \int_0^t kP_0\left(\frac{3V_0}{4\pi}\right)^{\gamma} dt \tag{9b}$$

$$\frac{R^{3\gamma+1} - R_0^{\ 3\gamma+1}}{3\gamma+1} \approx kP_0\left(\frac{3V_0}{4\pi}\right)^{\gamma} t \tag{9c}$$

$$R(t)^{3\gamma+1} \approx (3\gamma+1)kP_0\left(\frac{3V_0}{4\pi}\right)^{\gamma} t + R_0^{\ 3\gamma+1} \tag{9d}$$

Thus, Eq. (9d) represents the radius of the popcorn as a function of time. All other variables are predetermined constants. As was stated previously, γ is 1.3. P_0 is the initial pressure of the water vapor inside the kernel, which is approximately equivalent to the yield pressure constant 930,000 Pa. R_0 is the initial radius of the kernel, determined through many measurements to be approximately 0.003m. Finally, V_0 is the initial volume of the popcorn kernel before expansion. Substituting R_0 into expansion into the spherical volume formula, it can be found that the initial volume of a popcorn kernel is approximately $1.1309 \times 10^{-7}\,m^3$. Substituting all known values into Eq. (9d), an approximate equation for the radius of the expanding kernel with respect to time is determined to be as follows:

$$R(t)^{4.9} \approx 6.60\times10^{-4}kt + 4.34\times10^{-13}. \tag{10}$$

Solving for $R(t)$, we obtain the following:

$$R(t) \approx \left(6.60\times10^{-4}kt + 4.34\times10^{-13}\right)^{0.2041}. \tag{11}$$

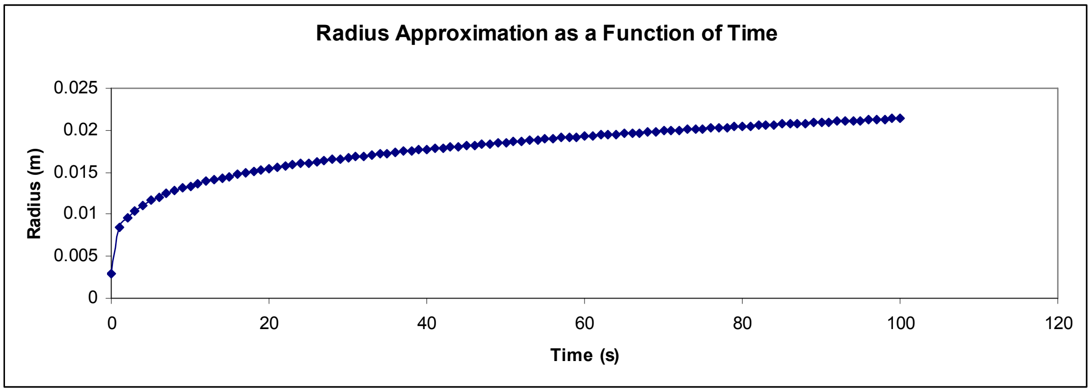

Figure 1: Through graphical analysis, the radius initially grows rapidly but soon slows down as it approaches an asymptotic limit. The *k* value used in this figure was chosen to be 0.0000001. The value of *k* does not affect the asymptotic limit. In this model, the atmospheric pressure of the system is kept at a constant of 30 inches of Hg below 1atm.

Figure 1 illustrates how the radius proposed by Eq. (11) increases with time until it reaches the asymptotic limit of approximately 2 cm. However, a more accurate graph of *R(t)* can be created by solving Eq. (8) using Euler's Method. With this approach, the initial assumption that the final pressure is insignificant compared to the pressure is removed.

Figure 2 shows that the results will be the same, with *R(t)* approaching an asymptotic value of approximately 1.8 cm. When the values of our constants are substituted into Eq. (8), the following expression is obtained:

$$\frac{dR}{dt} = k\left(\frac{5.65\times10^{-5}}{R^{3.9}} - 338.6\right)^{0.2041} \tag{12}$$

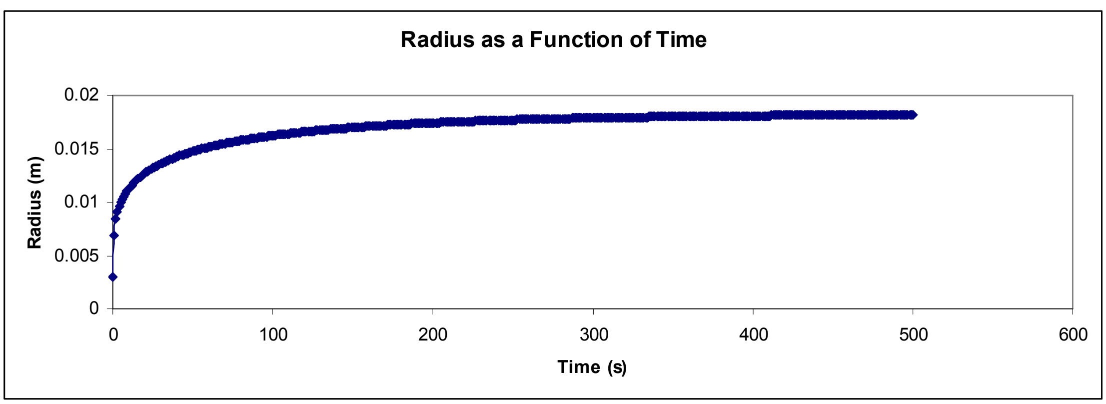

Figure 2: It can be seen from this graph that *R(t)* has a clear asymptote of approximately 1.8 cm. This demonstrates that the popcorn flake will not increase forever, but will hit a maximum radial length after a certain given time. The k value used in this model is 0.0000001, and the surrounding pressure of the system is kept at a constant 30 inches of mercury below 1atm.

The asymptote value can be found by setting Eq. (12) equal to zero,

$$0 = k\left(\frac{P_0\left(\frac{3V_0}{4\pi}\right)^{\gamma}}{R^{3\pi}} - P_f\right).$$

By substituting for R and the values of the constants, the asymptotic value for R is calculated to be 0.02285m at 30 inches of mercury below 1 atm. This value is reasonable for the flakes' size, in particular considering the simplicity of our model. Also, as stated previously, k plays no factor in deciding the maximum radial length. The proportionality constant only affects the rate at which the radius will increase.

PROCEDURE

To test our theoretical predictions, three different apparatuses were used: a stove popper, a movie popper, and a microwave. In each apparatus, the pressure was decreased at measured intervals to determine the effect of decreased pressure on flake size.

<u>Stove Popper</u>

In the first experiment, air was extracted from a modified pressure cooker using a vacuum pump. The effects of pressure on volume were determined by keeping temperature and popping time constant. The larger the final volume, the more successful the final pressure's effect would be in a given experiment. For this apparatus, the temperature was kept constant at 100^{o}C and time of twelve minutes. Twenty grams of popcorn kernels were utilized for each trial, each batch of kernels was counted, resulting in small variation from trial to trial.

After preparing the samples, a hot plate was set up at the highest available temperature setting. The pressure cooker was then placed on top of the hot plate as shown in Figure 3, and a thermometer directly attached to a PASCO computer interface was inserted into a special cavity through the top of the stove popper. The computer program, Data Studio, provided a temperature and time reading for each experiment.

Once the temperature reached 100^{o}C, a sample of kernels was poured into the pressure cooker, and the stopwatch on the Data Studio software was started. The lid on the stove popper was sealed shut and the vacuum pump was activated, effectively reducing the pressure to the desired amount below standard atmospheric pressure. Once all the kernels popped, the pressure was restored to safely remove the stove popper lid, and the stopwatch was stopped. The first run of each day was often employed as a preparatory trial in order to test and prepare the equipment. Therefore, the data from each first run was discarded. For all other trials, the popcorn flakes were removed from the stove popper and placed in a beaker where the total volume was measured. Additionally, calculations of σ, π, and ω were made using the total popped volume, the original sample weight, the number of un-popped kernels, and the original number of kernels from the individual experiments. Meanwhile, the pressure cooker was taken off the hot plate, and the lid was removed for a minute in order to cool the apparatus back to exactly 100^{o}C. This process was repeated four more times for at each reduced pressure value. All trials were completed over a 12 minute period. An average of the times was needed to pop all the kernels. Trials were run from 0 to 30 inches of mercury below atmosphere. From the data, average values of σ, π, and ω were calculated and graphed as a function of pressure below atmosphere. The results can be seen in Figures 6, 7, and 8.

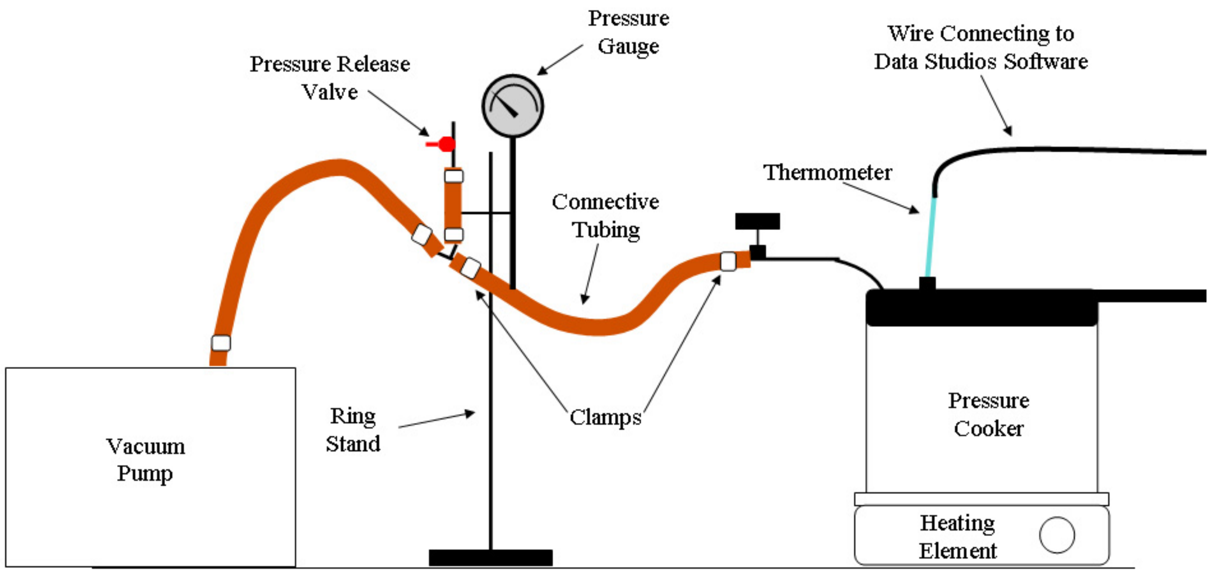

Figure 3: The stove popper apparatus.

<u>Movie Popper</u>

Another apparatus used to test our hypothesis was a popcorn popper similar to that used by the movie theater industry. The apparatus was modified as shown in Figure 4. The glass was removed and replaced with plexi-glass while the whole chamber was reinforced with steel and aluminum. Three pumps were attached to the apparatus to decrease the pressure inside the movie popper.

Pressure was the only variable in this experiment whose effect was tested on the flake's size and volume. The mass of the kernels used for each run was 85.0 grams. For each batch of 85.0 grams, the number of kernels was counted and recorded. Then the kernels were coated with oil so that the applied heat would spread out evenly and the popcorn would not burn. After loading the oiled kernels into the movie popper, the door was closed and tightly sealed. For the initial run at atmospheric pressure, none of the pumps were turned on. For later runs the pumps were turned on to lower the pressure. A release valve was used to control the pressure within the apparatus. Due to the size of the apparatus and the inefficiency of the pumps, the increments used were 5, 10, 15 and 19 inches of mercury below atmospheric pressure. For the lower pressures, metal support rods were inserted to prevent the apparatus from collapsing. Whenever a leak was detected, duct tape and silicone caulk were used to try and plug the leaks.

Once the desired pressure was reached, the apparatus was turned on to pop the kernels. When popping was completed, the movie popper and the pumps were turned off, and the valve was opened in order to return the apparatus to atmospheric pressure. All the popcorn was scooped out of the movie popper into a five-liter container. The volume was then measured and the unpopped kernels were counted. Using the collected data, the values of σ, π, and ω were determined in the same manner as that of the stove popper, using Eq. 3-5. The results can be seen in Figure 6, 7, and 8.

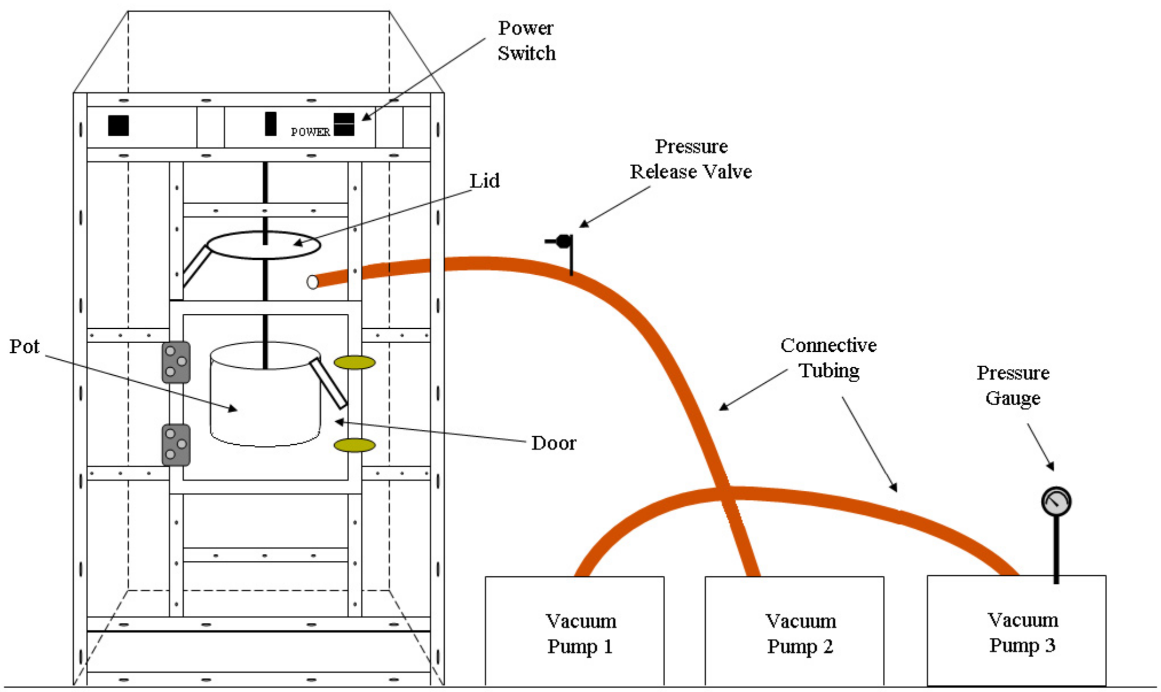

Figure 4: The movie popper apparatus.

Microwave

To set up the microwave popping apparatus, two hard plastic dog bowls were turned into a heating container for the popcorn. This container enabled the pressure surrounding the kernels to be controlled. A hole was drilled in one bowl and rubber weather stripping was placed around the edges of the other. Inside one bowl, a Teflon plate was placed to hold the kernels. A system of tubing was connected from a vacuum pump through the microwave to the dog bowl container inside. To measure and control the pressure in each trial, a pressure gauge and release valve were utilized. A diagram f the apparatus can be seen in Figure 5.

First, 10.0 grams of popcorn kernels were weighed out on an electric balance. The kernels in each sample were counted and then placed in the bottom half of a previously cut brown paper bag. The bag was partially sealed with glue, to prevent popping kernels from exiting. Next, the bag of kernels was placed in the dog bowl on the Teflon plate. The other dog bowl was placed on top of the first, and the two bowls were set inside the microwave. After inserting the rubber stopper and tube into the hole drilled into the dog bowl the vacuum pump was turned on to reduce the pressure inside the bowls. Once the desired pressure was reached, controlled by the valve, the popcorn was heated in the microwave for the optimized time of 3 minutes and 20 seconds. When the microwave was finished heating, the bottom bowl was removed. The top was left connected to the stopper and tube; we separated the unpopped kernels from the flakes. Finally, the inside of the microwave was cooled with wet paper towels to prevent overheating. This procedure was repeated five times for each pressure value. Pressure was reduced by increments of 5 inches of mercury from 0 to 30 inches of mercury below atmospheric pressure. Values of σ, π, and ω were calculated in the same manner as that of the previous two procedures.

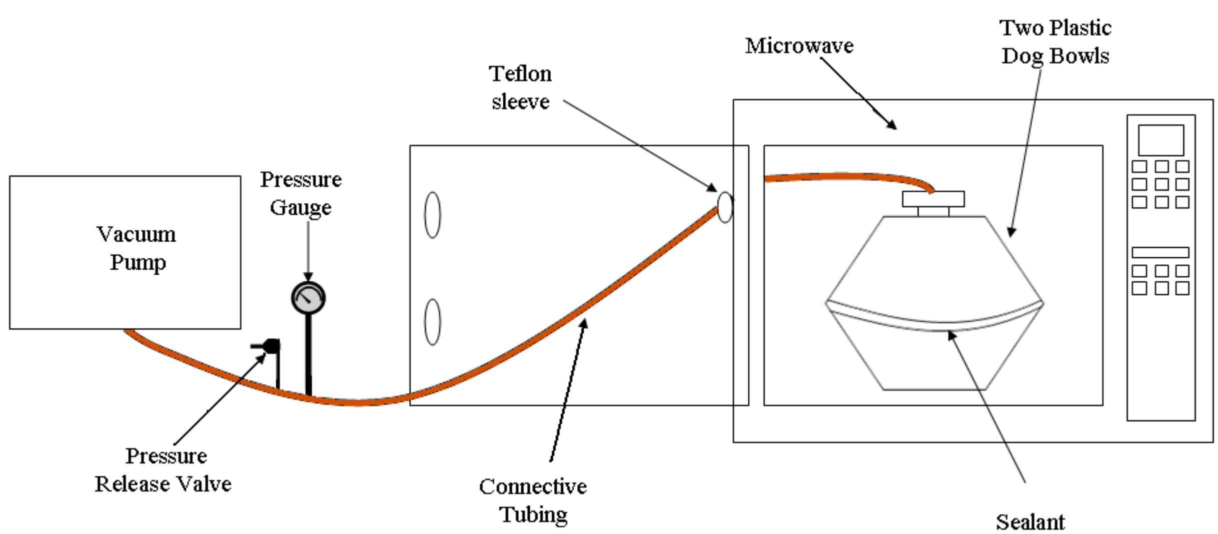

Figure 5: The microwave popper apparatus.

RESULTS

All three apparatuses displayed steady increase in the flake volume per unit mass (σ) as pressure decreased in the experiment. As can be seen in Figure 6a, the microwave apparatus had the highest value of σ for a given pressure. In the movie popper, only 19 inches of mercury below atmosphere could be achieved due to leaks in the apparatus. Overall, the values of σ at the lowest possible pressure increased significantly as compared to atmospheric pressure, as illustrated in Figure 6b. The same increasing trend occurred for flake volume (π) versus pressure, with the largest value of 9.8 cc/flake occurring at 30 inches of mercury below atmosphere for the microwave. These values and this trend can be seen in Figure 7a and 7b. When dealing with waste, the microwave and stove popper apparatuses proved to be much more efficient at lower pressures, as there was a significant decrease in waste, as can be seen in Figure 8. However, the percent waste for the movie popper remained fairly constant among the different pressures. This is because the movie popper is so efficient without reduced pressure, that lowering the pressure has almost no effect.

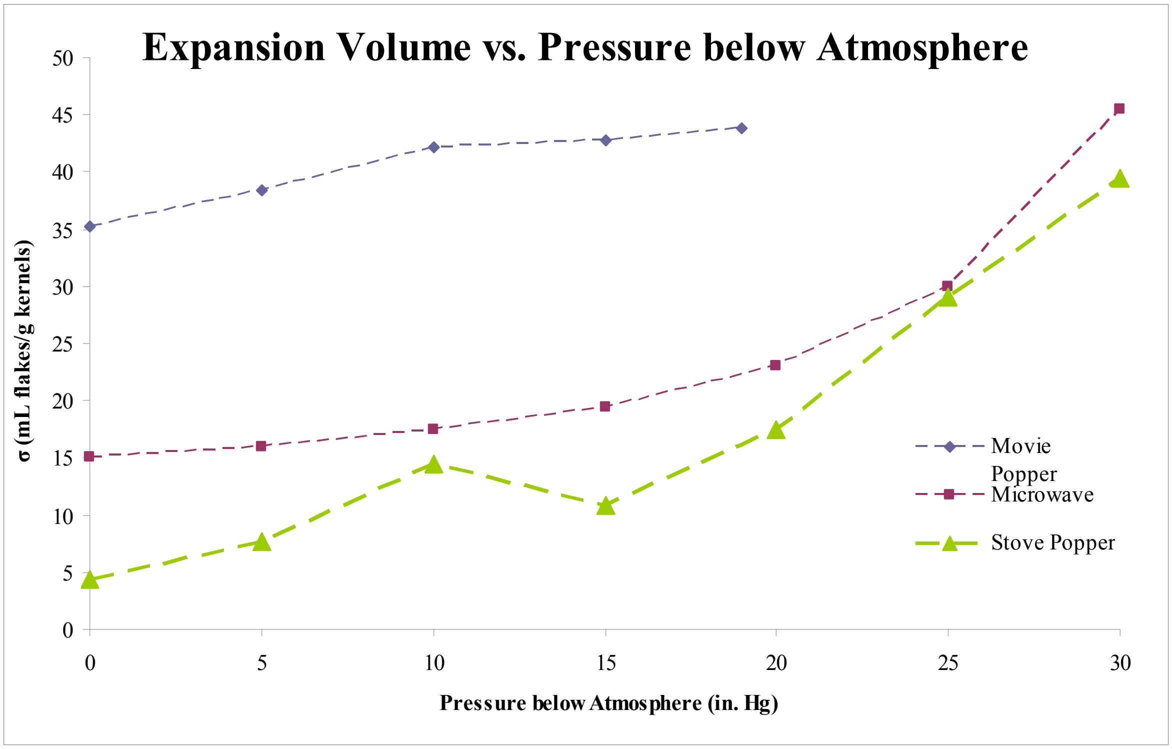

Figure 6a: Change in the expansion volume as a function of pressure below atmospheric pressure for all three apparatuses.

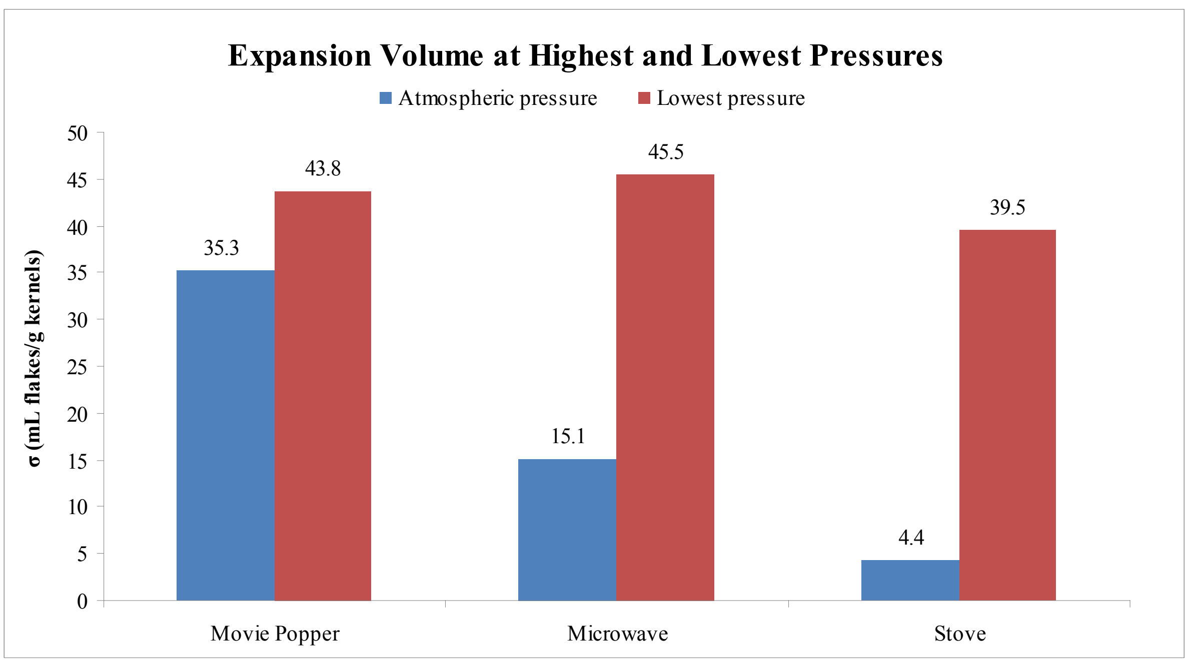

Figure 6b: Comparison of the results produced by the largest pressure decrease and those from atmospheric pressure.

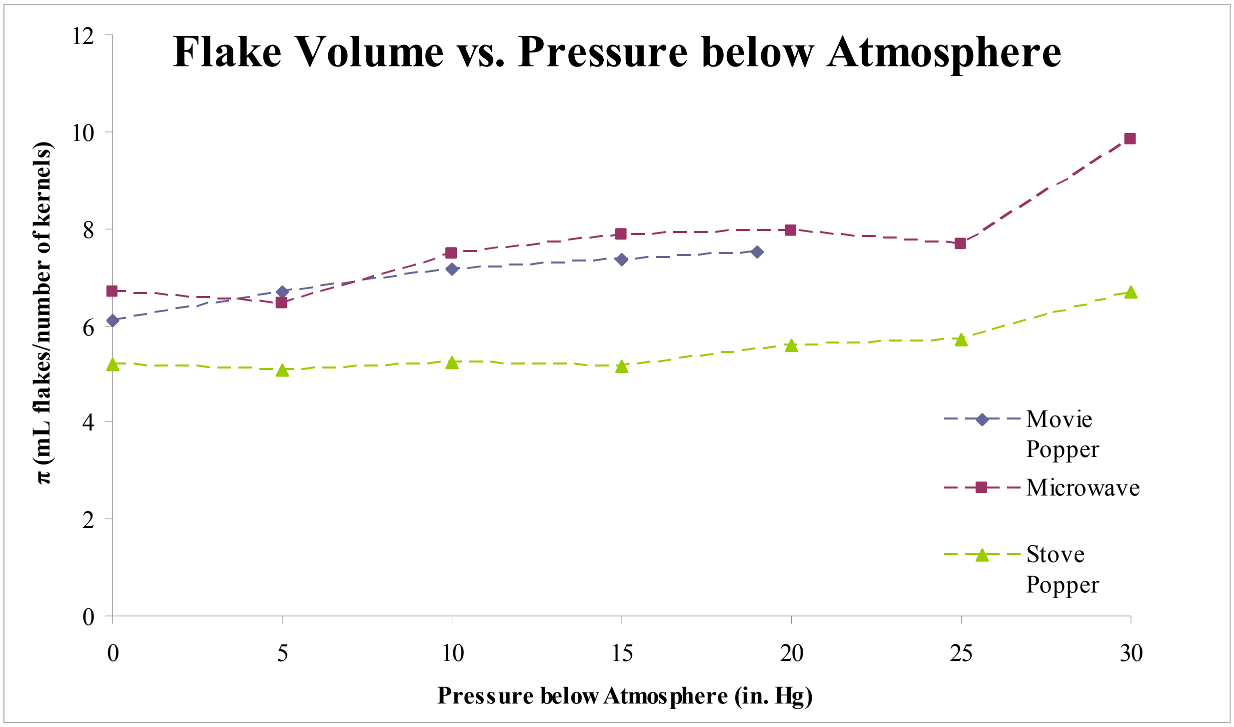

Figure 7a: Flake volume as a function of pressure below atmospheric pressure for all three apparatuses.

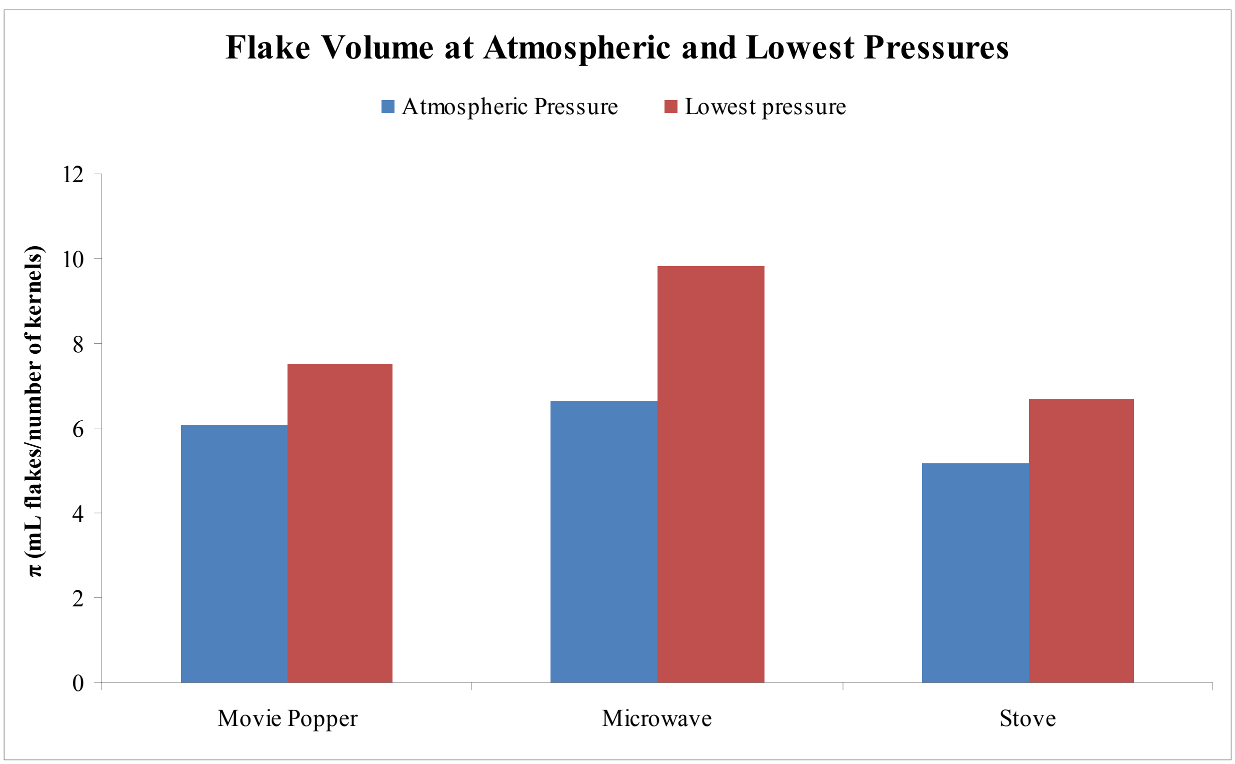

Figure 7b: Comparison of the results produced by the largest pressure decrease and those from atmospheric pressure.

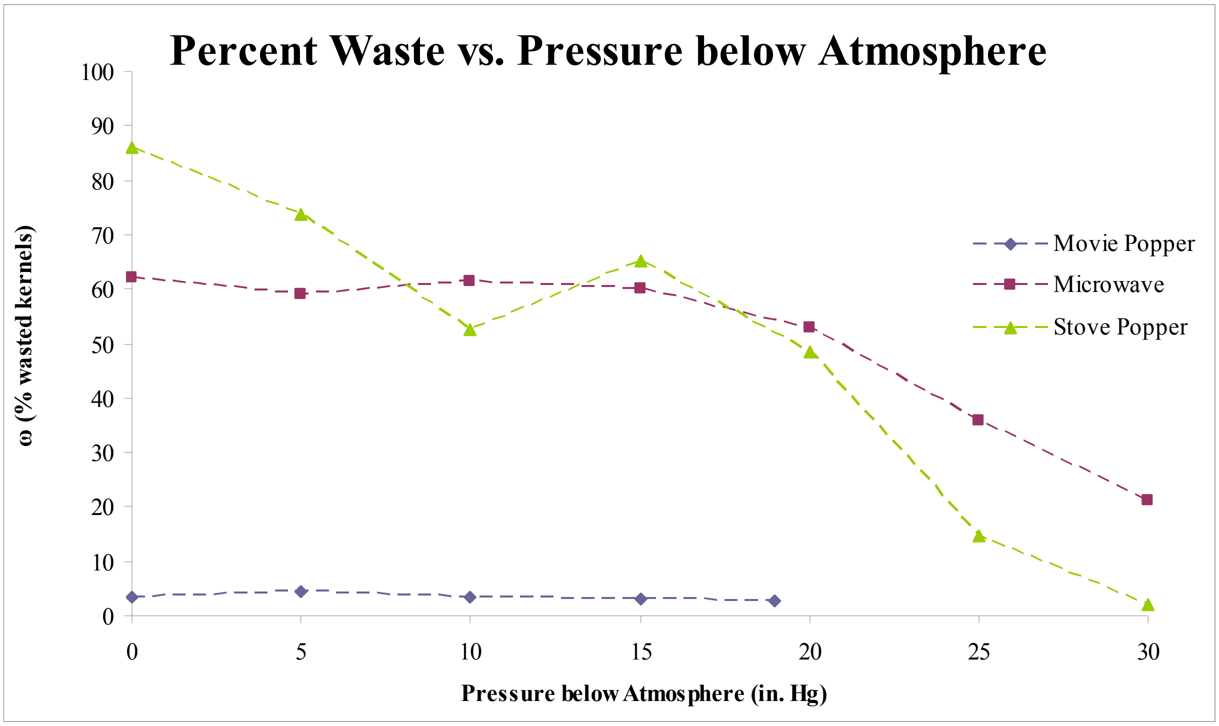

Figure 8a: Percent waste as a function of pressure below atmospheric pressure for all three apparatuses.

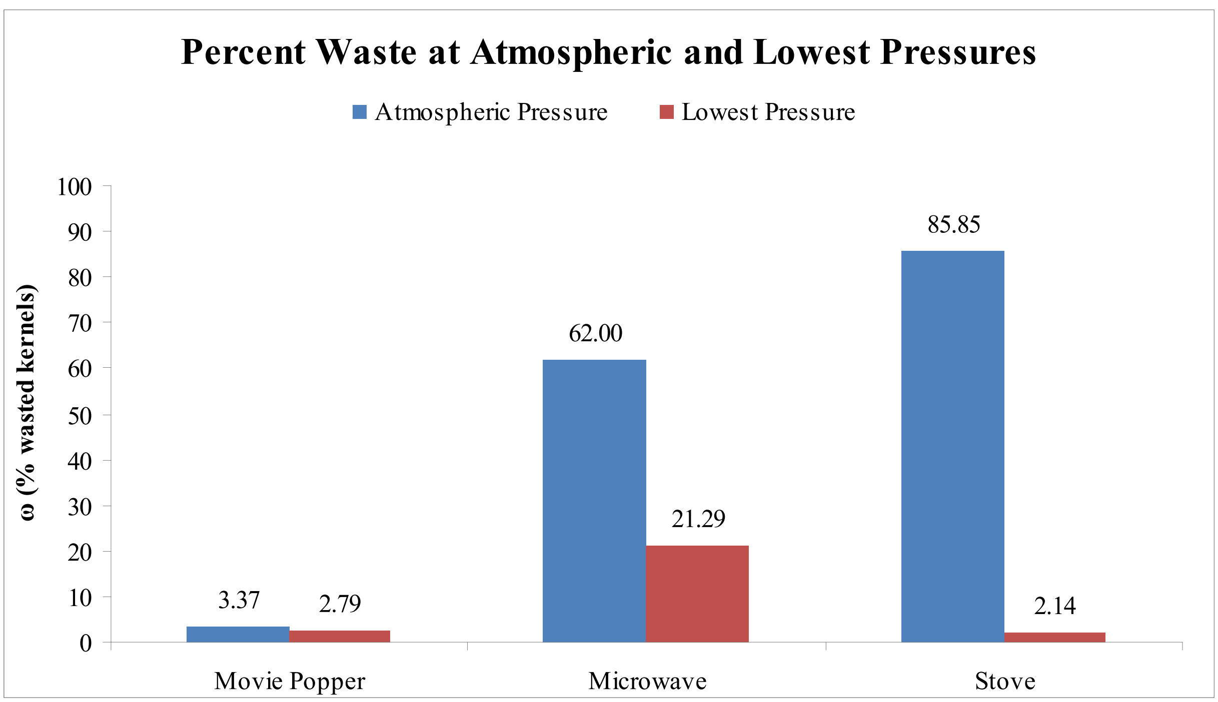

Figure 8b: Comparison of the results produced by the largest pressure decrease and those from atmospheric pressure.

DISCUSSION

The three apparatuses used during this experiment revealed that lowering the pressure in the popping chamber increases popcorn size, thereby optimizing the industry-defined variables σ and π while significantly reducing the waste, ω. Although the methods of popping varied amongst the apparatuses, the overall conclusions reached were similar.

Microwave

The results of the microwave popcorn apparatus showed that as pressure inside the microwave was lowered the volume of the popped corn increased. As the pressure increased in the microwave from a vacuum (30 inches of mercury below atmosphere) to atmospheric pressure, the expansion volume σ increased from 15 to 46. This shows how much more the flakes expanded in a vacuum than at atmospheric pressure, given a constant starting weight. When the pressure in the microwave decreased, flake size π also increased from 6.7 to 9.8, indicating a distinct increase in the average volume of each flake with reduced pressure. While both π and σ increased, the percentage waste, ω, decreased from 62.0 to 21.3. Clearly, decreasing waste produced occurred despite varying microwave temperatures, popcorn bag sizes, and microwave outputs.

Several errors were possible during the preparation and popping of the corn in the microwave that may have suppressed our increases. The first and perhaps most drastic source was the way in which the microwave was cleaned after trials, which involved wiping the inside of the microwave with a wet paper towel to prevent overheating. We attempted to keep the amount of water and number of towels we used constant. Yet, varying temperatures could have resulted since the microwave plate and surface would have different levels of heat. However, it was not possible to measure this effect. Other errors could be from the size of the popcorn bag, which at times was not sufficient to hold all the popped popcorn. As a result, some kernels may not have popped to their maximum size during the popping process. Finally, the output of the microwave, which tends to run at different degrees of power, varied as numerous trials were performed. As a result, there were occasionally large fluctuations in the amount of popped kernels during the popping process. Overall, waste was reduced and volume and popped flake volume were increased.

Movie Popper

The experimental results of the movie popper demonstrated a distinct increase in the expansion volume and flake size of the popcorn as pressure was decreased. Figures 6b and 7b show that the values for σ and π increased as was predicted. The value of σ increased from 35 cm^3/g at atmospheric pressure to about 44 cm^3/g at 19 in. Hg below atmospheric pressure. The increase in σ was drastic at first, but then slowed down significantly as pressure decreased. As shown in Figure 6b, the value of π also increased from about 6 cm^3 per flake to about 7.5 cm^3 per flake at the extreme pressures. This is only an approximate 7.7% increase in radius of the flakes, which would be hardly noticeable. However, when filled into our 5-liter bucket, the 85 grams of popcorn filled 3.0 liters at atmospheric pressure and 3.8 liters at reduced pressure, a noticeable difference. Figure 8a suggests that, waste percentage of kernels, ω, were statistically constant as pressure was lowered. The amount of oil used and the degree to which the kernels were evenly spread in the pot made the popping process very efficient despite the pressure. Therefore, we cannot establish a definite relationship between decreased pressure and ω for the movie popper. Ultimately, the results show that decreasing the pressure around kernels while popping popcorn leads to an increase in the values of σ and π.

<u>Stove Popper</u>

The stove popper results also indicated a clear relationship between popcorn volume and pressure. The smallest volume of popcorn was produced at normal atmospheric pressure, while a much larger volume of popcorn was produced at a nearly complete vacuum using a constant mass of kernels as shown in Figures 6 and 7. As such, the results exemplified an indirect relationship between popcorn flake size and pressure. The waste of popcorn drastically decreased as the pressure was lowered. One can see from Figure 8 that normal atmospheric pressure that, there was a high percentage of waste, while at 30 inches Hg below atmospheric pressure, there was little to no waste.

For most of the trials in the experiment, the results were consistent with the original hypothesis that decreasing pressure would increase popcorn production. However, there were errors that were possible due to some unreliable thermometer readings on the Data Studio software as well as the rough estimation of popcorn volume. The thermometer used was capable of reading the temperature inside the stove popper while keeping a tight seal to maintain internal pressure, but was unable to read any temperature greater than 100°C. Therefore, the temperature inside the pot for the duration of each trial was unknown. Since the starting temperature and total time for each trial remained constant, the results should not have been significantly affected, but an unknown changing temperature inside the cooking apparatus may be a source of error. Measuring the volume of popcorn proved complicated. A more accurate method of calculating volume, such as water displacement, was not practical in the case of popcorn, so beakers calibrated in intervals of 50 cc were used to estimate the volume of a certain set of trial results. The volume of each sample size was always rounded up to the next 50 cc interval and the overall trends in volume followed our expectations. However, inaccurate estimations of flake volume may still be a problem.

Overall, values for obtained by three apparatuses exceeded industrially accepted values and expectations. First and foremost, the microwave popper's σ value of 46 cc/g at a vacuum edged out industry's value of 45cc/g and the π value for the microwave of 9.8 cc exceeded the 8 cc of industry. This shows that the microwave popper produced significantly larger flakes compared to industry's popping apparatuses. The stove popper and movie popper beat industry's ω value at lower pressure, wasting only 2.79% and 2.14% kernels, respectively. Industry's standards showed a 6.8% waste in kernels, so the two apparatuses proved to be much more efficient. All in all, the microwave popper would greatly benefit the popcorn industry economically, as it produced larger flakes and larger flakes per unit mass, while helping minimized kernel waste. Additionally, as stated in the theory, the ideal radius of the popcorn flake popped at 30 inches under 1atm is 0.023m, or 2.3 cm. This shows that the orders of magnitudes of the experimental results are similar to the actual values, indicating the efficiency of popping at reduced surrounding pressure.

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APPENDIX A

An important aspect of the modeling popping of popcorn is the fact that it is an adiabatic process. An adiabatic process, also called an isocaloric process, is a process in which there is no heat transfer between the system and the surroundings. By combining this fact with the first law of thermodynamics:

$$dU = \Delta Q - \Delta W$$

the following expression can be found:

$$dU + \Delta W = 0\,. \tag{1}$$

In Eq. (1), dU represents the change in expensed internal energy while ΔW represents the change in amount of work done by the system. First, work must be redefined in terms of force. By definition,

$$W \equiv \int F ds\,, \tag{2}$$

where F is force and ds is the change in distance. Also by definition, pressure is equivalent to the ratio of force and area

$$P{=}F/A\ \ . \tag{3}$$

By rearranging Eq. (3), it can be found that the force is the product of the pressure and the area or

$$F{=}PA\ \ . \tag{4}$$

By substituting Eq. (4) into Eq. (2), we find:

$$W = \int PAds = \int PdV\ , \tag{5}$$

where dV is the change in volume. If the derivative of both sides of this expression is taken with respect of the volume, it can be found that

$$\Delta W = PdV\,. \tag{6}$$

Next, the amount of expensed internal energy is redefined in terms of pressure and volume. By definition,

$$U \equiv \alpha nRT\,, \tag{7}$$

where α is the degree of freedom divided by 2, n is the number of moles, R is the ideal gas constant, and T is the temperature. By taking the derivative of both sides with respect to temperature, the following expression is obtained:

$$\frac{dU}{dT} = \alpha nR\,, \tag{8}$$

which can be rewritten as

$$dU = \alpha nRdT\,. \tag{9}$$

From this point, *nRdT* is an expression that can be found in the ideal gas equation:

$$PV{=}nRT. \tag{10}$$

By taking the derivative of both sides in terms of temperature, it can be shown that

$$dPV + PdV = nRdT\,. \tag{11}$$

When Eq. (11) is substituted into Eq. (9), the change in the expensed internal energy can be redefined in terms of pressure and volume as follows:

$$dU = \alpha\left(dPV + PdV\right). \tag{12}$$

By substituting Eq. (6) and Eq. (12) into Eq. (1), the following relationship between the volume and the pressure of an adiabatic process can be found:

$$-PdV = \alpha dPV + \alpha PdV\,. \tag{13}$$

Using some calculus and algebra, the following steps are taken:

$$-(\alpha+1)\frac{dV}{V} = \alpha\frac{dP}{P} \tag{14a}$$

$$\int_{V_0}^{V}\frac{-\left(\alpha+1\right)}{V}dV = \int_{P_0}^{P}\frac{\alpha}{P}dP \tag{14b}$$

$$-\left(\alpha+1\right)\ln\left(\frac{V}{V_0}\right) = \alpha\ln\left(\frac{P}{P_0}\right) \tag{14c}$$

This leads to the expression

$$PV^{\frac{\alpha+1}{\alpha}} = P_0V_0^{\frac{\alpha+1}{\alpha}} \tag{15}$$

By substituting the constant γ for its equivalent $\dfrac{\alpha+1}{\alpha}$ where γ is the ratio of specific heats, it can be found that

$$PV^{\gamma} = P_0 V_0^{\gamma}.$$

APPENDIX B

Data Collected from the Stove Popper Apparatus			
Pressure (in. Hg below atm)	Expansion Volume, σ (cc/g)	Flake Size, π (cm3)	w (wasted/original) x 100%
0	4.10	4.71	85.9
5	7.70	5.06	73.8
10	13.0	5.14	54.7
15	11.3	5.49	65.2
20	18.0	5.66	48.6
25	30.0	5.89	14.6
30	40.0	6.78	2.14

Table 1: The average values for σ, π, and ω of popcorn popped under various pressures in the microwave.

Data Collected from the Movie Popper Apparatus			
Pressure (in. Hg below atm)	Expansion Volume σ (cm^3/g)	Flake Size π (cm^3)	Un-popped Kernels ω (%)
0	35.3	6.09	3.37
5	38.3	6.70	4.49
10	42.1	7.17	3.36
15	42.8	7.17	3.36
19	43.8	7.50	2.79

Table 2: The average values for σ, π, and ω of popcorn popped under various pressures in the movie popper.

Data Collected from the Microwave			
Pressure (in. Hg below atm)	Expansion Volume σ (cm^3/g)	Flake Size π (cm^3)	Un-popped Kernels ω (%)
0	15.1	6.67	62.0
5	16.0	6.46	59.1
10	17.4	7.48	61.4
15	19.4	7.85	59.9
20	23.0	7.95	52.9
25	30.0	7.67	36.0
30	45.5	9.83	21.3

Table 3: The average values for σ, π, and ω of popcorn popped under various pressures in the microwave.